Common Proportion Error Problem Perspective and Value Analysis

Yu Huan

Capital Normal University, College of Education, Beijing, 100037, China

Keywords: Mathematics proportion error, Proportion of thinking, Proportional reasoning, Common proportion error, Isomorphism problem

Abstract: It is one of the core tasks of mathematics teaching reform for teachers to understand the thinking law of students. This paper takes proportion as the research object, combs the common errors of students, analyzes the common solutions of proportion problems, explains the reasons of students' errors from the perspective of knowledge, program strategy and intuition, and finally obtains the reasons that children's understanding of concentration is limited, strategies are improper, multiplication relationship is not recognized, intuitive influence and other reasons lead to children's errors. On this basis, according to the proportional reasoning model, judge the level of students' proportional reasoning, and make contribution to improve students' proportional reasoning ability.

1. Introduction

Ratio and proportion is an important part of primary school mathematics. To solve the price problem in primary school, it is expressed that total price and quantity are two kinds of related quantities. The total price varies with the quantity and the ratio with the corresponding quantity is always constant, price = total price/quantity, which can be regarded as a special proportion problem. Another problem in elementary school, speed = distance/time, can also be seen as a special proportion problem; The grade equivalence relation can also be regarded as a problem of proportion. From the simple proportion problem in primary school to the linear model in middle school to the calculus in college, there are all kinds of proportion reasoning and linear thinking. Proportional reasoning is a kind of thinking mode used by students to solve the problem of proportion, which is very important for students' mathematical learning.

In the document of *National Council of Teachers of Mathematics (NCTM),* it is pointed out that this is an important content, which is worth spending time and efforts to ensure the development of students' proportional reasoning ability[1]. *Mathematics Curriculum Standards for Compulsory Education (2011)* points out that the development of reasoning ability runs through the whole process of mathematics learning. Reasoning is the basic way of thinking in mathematics, and it is also the way of thinking frequently used in people's study [2]. Thus, it can be seen that proportional reasoning ability is crucial to students' mathematical learning. However, the ratio and proportion problem are considered as a complicated and high error rate problem in elementary mathematics. There are so many students' errors that teachers can not understand the causes of their common mistakes. Therefore, the research on common errors of ratio and proportion and how teachers analyze students' thinking patterns have become the top priority.

2. Perspective on Scale Problem

The content of ratio and proportion appears in the textbook of the sixth grade of elementary school mathematics in the people's education edition. It is a kind of new knowledge to understand the relationship between quantity and solve practical problems with a new point of view, a new perspective and a new method introduced by the content of division and fraction that has been learned. The characteristics of ratio and proportion are to understand and study the relationship between quantity from the relationship between the movement and change of things. In the second edition of *The Dictionary of Mathematics* compiled by wang yuan, ration is defined as: to compare the multiple relation between two similar quantities to become the ratio of the two similar quantities.

  DOI: 10.25236/ietrc.2020.120

The comparison of two quantities can also be described as the division of two quantities. The ratio of a to b is denoted as a:b or a/b. And a:b, where b is not equal to 0, is called the ratio[3]. The content distribution of this unit is mainly a preliminary understanding of the ratio, the basic properties of the ratio; The meaning of proportion is explained in *Shuo Wen Jie Zi · Ren Bu*, which is not a restriction case of ratio, but a combination of two identical meanings. The metaphor behind is that two ratios are the same[4]. The algorithm of proportion is recorded as early as in chapter 2 of *the Nine Chapters on the Mathematical Art*, which focuses on the ratios of various grains and the algorithm of proportion[5]. The most famous scaling algorithm has four items, one of which is unknown[6]. The proportions of the textbooks in this unit is mainly research the meaning, the proportion of the proportion of basic properties, solution, direct ratio, the inverse proportion of these knowledge, and use this knowledge to solve practical problems such as scale, graphics, enlarged and reduced, infiltration function of ideas, for the students in the future middle school mathematics learning foundation. Please see table 1.

Table 1 Contents Distribution of the Ratio and Proportion of the Teaching Edition

Grade	Unit	Unit Name	Specific Content
Volume1,grade6	Unit4	Ratio	A preliminary understanding of the ration, the basic nature of the ratio.
Volume2,grade6	Unit4	Proportion	The meaning of proportion, the basic properties of proportion, the solution of proportion, direct proportion, inverse proportion, scale, enlargement and reduction of figures, and solving problems with proportion.

Here is a simple scale problem. What we know: two cups of salt water in cup a and cup b. There are 6 tablespoons of salt in cup a and 14 tablespoons of water. Now the water has been increased to 21 tablespoons. ________tablespoons of salt are added to make the concentration of cup b the same as cup a. A. 27 B. 13 C. 9 D. Not Sure. Common practices of students are as follows:

Table 2 Solution Process

Student	Answer	Process
M	A.27	Tablespoons of water 14 tablespoons of water, 21 tablespoons more, so 21 tablespoons more salt, 6+21=27 tablespoons, 27 tablespoons of salt.
N	B.13	Tablespoons of water 14 tablespoons of water in the cup, increase to 21tablespoons, that means 7 tablespoons more, so 6+7=13tablespoons, add 13 tablespoons of salt.
O	C. 9	Tablespoons of water expanded 1.5 times from 14 to 21, so the salt expanded 1.5 times.
P	C. 9	Tablespoons 6/14=X/21=3/7, X=9, so add 9 tablespoons of salt to make the same concentration in both cups.
Q	C. 9	Tablespoons 6/X=14/21=2/3, X=9, so add 9 tablespoons of salt to make both cups equally salty.
R	C. 9	Tablespoons 6/14=3/7, 3/7 times 21=9, so we need to add 9 tablespoons of salt.
S	D.Not Sure	There are 6 teaspoons of salt in cup a, 14 teaspoons of water, and 21 teaspoons in cup b.
T	D.Not Sure	Existence of a proportional relationship, 6/14 =21/ D, D =44, can not determine the answer.

The above is the solution process of 9 pupils to the concentration problem. Follow table 2. Next, the article will use common practices that students are familiar with, dig behind the big ideas, explore how students think and the causes of errors.

3. Common Errors

This is a very common problem in elementary mathematics proportion study. Given three of the four variables a,b,c solve for the other variable. Please see table 3. Studies show that in 1983,Vergnaud defined such problems as isomorphism of measure problems [7],in the same reasoning system, there are similar structures and corresponding elements, and there are two variables of linear function relationship.

Table 3 Problem Analysis

Salt	Water		Measurement1	Measurement2
6	14	Value1	a	b
?	21	Value2	d	c

In the face of different students' error algorithms, teachers explain that students are careless, sloppy, not serious, etc. Through the common error phenomenon, whether there are other reasons for students' wrong solutions, whether students can dig deep thinking rules, analyze students' wrong ideas.

3.1 The Relation of Language and Mathematical Symbols.

From an intuitive point of view, students see increase and want to use addition, see decrease and want to use subtraction, see than... If you add more, you'll see more than... Less use subtraction, see total want to use addition, see the left just want to use subtraction, see fly to use addition, see the away to use subtraction, see brought want to use addition, see the away want to use subtraction, and so on, professor Gao Shuzhu once on children's intuition thinking and error are studied, put forward the one to one correspondence intuition[8]. There is a one-to-one correspondence between the expressions of language and mathematical symbols in the problems mentioned above, and students often hope to achieve the correspondence between language and mathematical symbols in the questions.

M and N, seeing the increase in the problem, used the method of addition to different degrees. There is such a problem in the problem solving teaching of the junior grade of primary school, there are some swans on the lake, fly away five, still remain eight, question: How many swans ______. The teacher expected the students to write 5+8=13, but the students wrote 13-5=8 because of the two eye-catching words in the question, one is fly away, the other is still left. Written language and mathematical language correspond to each other in principle. Intuitively speaking, students prefer to use subtraction to add.

3.2 The Law of Intuition Same a-Same B

In this problem, there are two cups of salt water in the first cup, of which there are 6 teaspoons of salt and 14 teaspoons of water. Now the water is increased to 21 teaspoons[9]. M students directly add 21 to calculate. If you increase the water to 21, you have to increase the salt by 21, which is to keep the concentration the same, the same water, the same salt, 21 more water, 21 more salt, and you get the wrong answer of 27. N students' answers also apply the thinking mode of superposition reasoning. N students get 21 scoops, which is actually an increase of 7 scoops, so you have to add 7 scoops to get the salt, the same salt with the same water, 7 scoops of water, 7 scoops of salt, 13 scoops of wrong answer.

3.3 The Law of Intuition More a-More B

From a concept-based perspective, the following knowledge is needed to solve this problem:

(1) To understand the concept of the same concentration;

(2) To determine the specific ratio identity between the number of salt scoops and the number of water scoops; to determine the specific ratio identity between the number of salt scoops and the number of water scoops in the same water cup.

(3) The similarity ratio is used to express the proportional relation

(4) For the concentration problem, the first thing to understand is the same concept of concentration.

S student's answer is not sure, he has identified a cup of salt water in the six teaspoons of salt, 14 water scoop, and b cups have 21teaspoons of three different quantity, but his mind was in a glass with 6 + 14 = 20, b cup already has 21, if party a copies of the cup and b cup's score is even, then (−1) + 21 = 20, means to come up with a from b cup. It is not common sense, so the choice is uncertain. This response suggests that the students did not understand the concept of concentration and intuitively got the number of servings per cup. According to the explanation of More a-more B intuitive law, it can be known that the More shares students make, the higher the concentration, the fewer shares, the less concentration, and the equal shares, the equal concentration. For another example, the following table shows the responses of children aged 5-8, as shown in table 4, which reflects the misunderstanding caused by students' intuition.

Table 4 Error Caused By Intuition

Question	Error	Reason
Two identical cups, the first cup with 5 parts water and 5 parts salt; Put 1 part water and 1 part salt in the second cup. _____ cup is saltier.	The first cup is more salty.	The more salt the cup, the saltier; More cups of water and salt, more salt; The concentration in the first cup was five times that in the second cup;
I'm going to take two cups of salt water and I'm going to pour it into the larger third cup, and I'm going to have six parts of salt and six parts of water, and what happens to the salt water at this point?	The water is saltier than it used to be.	Six parts of salt, six parts of water is more than two cups of salt water. As a result, the water became saltier.

This is More A-More B intuition rules that lead students to make mistakes. On the contrary, students M, N and S Error. From the perspective of knowledge, they do not understand or have a limited understanding of the concept of the same salinity, and do not understand the relationship between the number of spoons of salt, the number of spoons of water and the same salinity. If the spoon number is the same, then the salt content is the same. No relationship between quantities was found. In fact, the number of spoonful of salt and the number of spoonful of water increases or decreases proportionally to maintain the same saltiness, rather than simply adding a few spoonful to one and a few spoonful to the other.

3.4 Wrong Explanation of Procedure Policy

Speaking from the process-based perspective, students need two basic problem-solving strategies to solve this problem: first, find the constant value; The second step is to cross multiply. The way P students do it is 6/14=X/21=3/7, X=9, find the constant ratio first, then look for the missing value by intra-group ratio. R students 6/14=3/7, 3/7×21=9, through group ratio, find the constant value first, then do multiplication, look for the missing value. And what they did was they looked for a constant value between the groups, and they cross-multiplied to get the value of X, 6 over X is equal to 14/21, X is equal to 9. What T students did was to find that there was a proportional relationship 6/14=21/d, and d=44. However, due to the wrong solution strategy, the correct missing value was not found.

4. Develop Students' Proportional Reasoning Thinking

Understanding proportions begins with the ability to understand multiplicative relationships, distinguishing them from additive relationships. The key to realizing the transition from additive reasoning to multiplicative reasoning lies in a sufficient understanding of the invariant relationship between variables in two sets of similar concepts. Formally, the ratio can be expressed as a fraction. For example, a/b=c/d. There are two kinds of different variables in the problem, namely the amount of salt and the amount of water. The two variables have a covariation relationship, that is, the relationship of interdependence and collaborative change[10]. two interrelated quantities, one of which changes and the other also changes. If the ratio of the corresponding two quantities in these two quantities is constant, they are in a positive proportional relationship.

Baxter and Junker (2001) proposed a model of five developmental stages of proportional reasoning, from qualitative understanding of quantitative relations, quantitative attempt super position reasoning, recognition of multiplicative relations, covariant relations and invariance, and finally gradually developed to functional relations. The five stages were progressive from the lower to the higher, as shown in table 5.

Table 5

Development Stage	Characteristics	Strategy
Qualitative	Recognize the concept of quantity, will not calculate or use the illogical calculation.	Invalid Intuitive
Early Attempts at Quantifying	Attention is paid to the difference between the given quantities, but not to the constant ratio. Using superposition reasoning, try to find missing values.	Additive Reasoning
Recognition of Multiplicative	Intuitively aware that there is a proportional relation in the problem situation, but the relation cannot be expressed correctly or the calculation is incorrect.	Proportion Attempt
Accommodating Covariance and Invariance	Begin to develop collaborative change relationship. Ratios can be done using basic arithmetic. There is informal doubling consciousness.	Valid Informal
Functional and scalar relationship	It has good cooperative change relation and invariance concept. Look for ratios in a cup of salt water. The proportion relationship between two cups of brine was found, and the missing value was found by intra-group comparison. Must find a fixed ratio, using the ratio to find the missing value.	Within Ration Between Ration (Functional) Scale Factor

Specific view, qualitative understanding of the specific performance for the students from the perspective of visual cognition to a variable, but there is no effective strategy can not be sure answer, S students identify a cup of salt water in six teaspoons of salt, 14 water scoop, and b cups have 21 spoon, but did not resolve this problem, not sure the answer, so S students in one stage.

The quantitative attempt phase is characterized by a focus on differences between the given quantities, but not on constant ratios. For example, use superposition reasoning to try to find missing values. Student M thought it was 27 scoops. He found that: there were 14 scoops of water in the cup, which added 21 scoops. N student's answer is 13 teaspoons, 14 teaspoons of water in the glass, increase to 21, that means increase by 7, so 6+7=13, add 13 salt; Neither of these solutions is correct. The solution is additive reasoning. Therefore, both M and N students are in the second stage of the model.

T students intuitively realized the existence of proportion relation, identified the multiplication relation, and realized the transformation from additive reasoning to multiplicative reasoning. They tried proportion, but unfortunately the calculated result was not correct. 6/14 =21/d, d=49. Therefore, the errors of T students are in the third stage of the development model.

The covariation relationship and invariance stages indicate that students begin to develop invariable multiplicative changes, ratios that can be used in basic arithmetic operations, and informal doubling awareness. O the students found that the water was 1.5 times larger from 14 to 21, so the salt was 1.5 times larger, so 9 tablespoons more salt. O students have an informal, effective awareness of doubling, and find a 1.5-fold relationship between the twofold saline, so O students are in the fourth stage of the proportional reasoning development model. O student's answer is correct, but does not list the formal proportional relation.

The fifth stage is the stage of function and scalar relation. Students have good concepts of quantity and invariance of collaborative change. Students P, Q and R used the known conditions to find the missing value correctly. Student P's 6/14=X/21=3/7, X=9, add 9 tablespoons of salt to make the two glasses equally salty as student Q did differently. Student P started with the ratio of salt to water in one cup, that is, the in-group ratio.

Student Q 6/X=14/21=2/3, X=9, add 9 tablespoons of salt, from the two cups of salt and water, make the two cups the same salinity, that is, the ratio between the groups, the thinking process and the answers are correct.

Student R's answer was 6/14=3/7, 3/7×21=9, so you need to add 9 tablespoons of salt to achieve the same salinity in two cups. Student R's approach is to find a scaling factor of 3/7, and then use 3/7×21=9 to get the answer.

5. Conclusion

Teachers should believe that students' misunderstanding is not completely wrong or useless. It is an important part of children's mathematics learning and development, and has a certain application value in teaching. According to the common errors in the proportion of students, teachers should dig deep into the causes of errors, explain the causes of students' errors from the perspective of knowledge, program strategy and intuition, and finally conclude that children's understanding of concentration is limited, strategies are improper, multiplication relationship is not recognized, intuitive impact and other reasons lead to children's errors. On this basis, according to the proportional reasoning model, judge the level of students' proportional reasoning, and make contribution to improve students' proportional reasoning ability.

References

[1] NCTM. Curriculum and Evaluation Standards for School Mathematics. Reston, VA: National

[2] Council of Teachers of Mathematics.1989.

[3] Ministry of Education of the People's Republic of China. Mathematics Curriculum Standards for Compulsory Education 2011 Edition [S]. Beijing: Beijing Normal University Press.2012.1:6-7.

[4] Wang yuan. Mathematical Dictionary (2nd edition) [M]. Beijing: Science Press.2018.12.

[5] Xu shen. Shuo Wen Jie Zi [M]. Beijing: ZhongHua Book Company.1963:167.

[6] Gao shuzhu. Mathematics Teaching in Primary School [M]. Shanghai: East China Normal University press.2015.7:37.

[7] Zhang cang etal. Translation by Zeng Hailong. Nine Chapters on Arithmetic [M]. Nanjing: Jangsu People's Publishing House.2011, 328-29.

[8] Vergnaud, g. Lesh & m. Landau. Acquisition of Mathematical Concepts and Processes[M]. New York: Academic Press.1983.127-174.

[9] Gao shuzhu. Perceptual Law in Thinking of Junior Students [J]. Mathematics Teaching Monthly · Primary School Edition.2017.11:4-6.

[10] Stavy, Tirosh.D. Intuitive Rules in Science and Mathematics: A Reaction Time Study [J]. International Journal of Mathematics Education in Science and Technology.2006,37(8):913-924.

[11] Gao shuzhu. Where to Calculate Decimal Multiplication [J]. Monthly Teaching Primary School Mathematics. 2018.09:4-7.

[12] Ji-won Son. How Preservice Teachers Interpret and Respond to Student Errors: Ratio and Proportion in Rectangles[J].Educational Studies in Mathematics.2013.9:49-70.