An Experimental Method for Determining the Viscosity Coefficient of Air

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Abstract: Starting from Millikan's oil drop experiment, this paper points out the problems existing in the current basic charge determination method, that is, the viscosity coefficient of air is not corrected and the influence of air buoyancy is ignored. In this paper, after adding these two factors, the expressions of the basic charge and air viscosity coefficient are deduced. Through actual measurement and calculation, the results show that the error of the basic charge is within 2%, and the experimental value is relatively close to the theoretical value in line with expectations. The error between the measured value of air viscosity coefficient and the theoretical reference value is within 4%.

1. Introduction

The discovery of charge discontinuities and the precise determination of electron charge have advanced understanding of microstructure [1]. Millikan's oil drop experiment was first designed and completed by experimental physicist Millikan, and it is a very important experiment in the development history of modern physics. It proves that the charge carried by any charged body is an integer multiple of a minimum charge-basic charge; it clarifies the discontinuity of charge; and accurately determines the value of the basic charge, which provides the basis for the experimental determination of some other basic physical quantities possibility. Millikan has been working on the measurement of the electric charge of tiny oil droplets since 1906. It took 11 years and after many major improvements, the conclusive experimental data of more than a thousand oil droplets finally confirmed the quantum nature of electric charge irrefutably. The electric quantity e = $(1.5924\pm 0.0017)*10^{-19}$ C of the basic charge is measured, and the modern accurate value is $1.602*10^{-19}$ C. The results of Millikan's oil drop experiment show that the charge of all charged objects changes discontinuously, and the charge carried by them is an integer multiple of the basic charge *e* of a certain minimum charge [2]. The significance of Millikan's oil drop experiment is to reveal the quantum nature of the electrical structure of matter, which plays a decisive role in people's understanding of the basic structure of matter, and has played a very important role in the history of modern physics [3], and won the Nobel Prize in Physics in 1923. Millikan's oil drop experiment is ingenious in design, simple in method and accurate in conclusion. The experiment also plays a great role in improving students' design thinking and experimental skills. In this paper, a new viscosity coefficient measurement method is proposed based on the Millikan oil droplet experiment, and the experimental verification is carried out.

2. Measurement Principle

In the university physics experiment, the purpose of Millikan's oil drop experiment is to determine the electric charge value of electrons by measuring the electric charge of the oil droplet, so as to verify the quantization of electric charge. There are two methods to measure the electric charge of oil droplets: one is the equilibrium measurement method, and the other is the dynamic measurement method. Because of its simple measurement principle, relatively easy operation and simple data processing, the balance measurement method has become a commonly used experimental method in university physics experiments. The specific measurement process of the balance measurement method is to use a sprayer to spray oil droplets between the two horizontal

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and parallel plates of the parallel plate capacitor, and set the distance between the plates as *d*. In the process of spraying and dispersing oil, the oil droplets are charged due to friction [4]. Assume that the mass of the oil droplet of the research object is *m*, the charged quantity is *q*, the voltage between the two polar plates is *U*, which can be adjusted, the gravitational acceleration is *g*, the electric field strength is *E*, and the air buoyancy is *F*. Since the density of air is much smaller than that of oil, the air buoyancy on the oil droplets can be neglected temporarily, so the oil droplets are only affected by the force of gravity and the electric field in the electric field (the central area can be approximated as a uniform electric field) in the parallel plate capacitor. Set the positive and negative of the two polar plates of the parallel plate capacitor, so that the two forces on the oil droplets are in opposite directions, and adjust the voltage value between the two polar plates of the parallel plate capacitor, so that the electric field force on the oil droplets is equal to the gravity to achieve a balance [5]. As shown in figure 1.

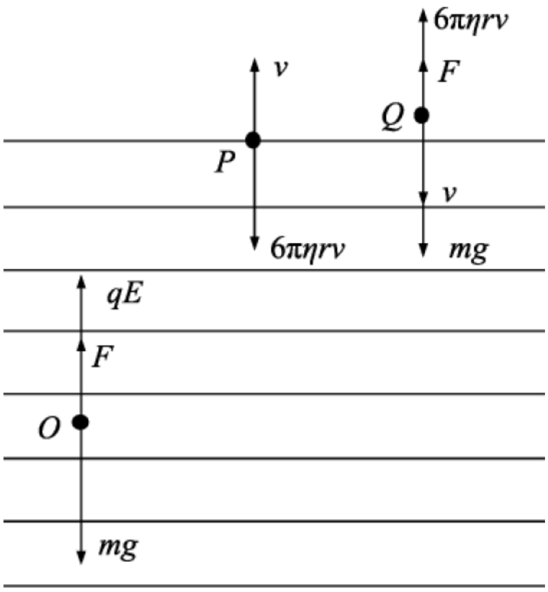

Figure 1 Schematic diagram of the force of oil droplets

By adjusting the voltage *U* between the plates, the two forces acting on the oil droplets can reach a dynamic balance. Then there are

$$mg = qE = q\frac{U}{d} \tag{1}$$

It can be seen from formula (1) that in order to measure the electric quantity *q* carried by the oil droplet, in addition to measuring *U* and *d*, the mass *m* of the oil droplet also needs to be measured. Since *m* is very small (approximately$10^{-15}\,\mathrm{kg}$), special methods are required to determine it. Oil droplets are generally spherical under the action of surface tension. Suppose the density of oil is ρ, the radius of an oil droplet is *r*, then the mass *m* of the oil droplet can be expressed as follows

$$m = \frac{4}{3}\pi r^3 \rho \tag{2}$$

When no voltage is applied to the parallel plates, the oil droplets are accelerated and descended by gravity, but because the viscous resistance *f* of the air to the oil droplets is proportional to the speed *v* of the oil droplets, the oil droplets drop for a certain distance, and after reaching a certain speed, the resistance *f* and The gravity reaches equilibrium (air buoyancy is negligible), and the oil droplet will descend at a constant speed. According to Stokes' law

$$f = 6\pi r \eta v = mg \tag{3}$$

In formula (3), η is the viscosity coefficient of air, and the size of the oil droplet radius obtained from the formulas (2) and (3) is

$$r=\sqrt{\frac{9\eta v}{2\rho g}} \tag{4}$$

The calculation formula of the electricity charged by oil droplets used in many textbooks is solved by the oil droplet radius formula (4) before correction. Such correction is incomplete and will bring systematic errors to the measurement results. For a small ball with a radius as small as 10^{-6} meter, the viscosity coefficient of the air should be corrected, and the Stokes law should be corrected at this time as

$$f=\frac{6\pi r\eta v}{1+\frac{b}{rP}} \tag{5}$$

where b is the correction constant and P is the atmospheric pressure. According to the revised viscous resistance formula, the oil droplet radius is

$$r=\sqrt{\frac{9\eta v}{2\rho g}\frac{1}{1+\frac{b}{rP}}} \tag{6}$$

(6) The speed v of the oil droplet falling at a constant speed in the formula can be measured by the following method: when the parallel plate is not applied with voltage, the time t used for the oil droplet to drop by l length is measured, that is

$$v=\frac{l}{t} \tag{7}$$

After calculating the formula (6), we get

$$r^2+\frac{b}{P}r=\frac{9\eta v}{2\rho g} \tag{8}$$

The solution of equation (8) is:

$$r=-\frac{b}{2P}\pm\frac{1}{2}\sqrt{\left(\frac{b}{P}\right)^2-\frac{18\eta v}{\rho g}} \tag{9}$$

Since r is the radius of the oil droplet and cannot take a negative value, the solution of equation (7) is

$$r=-\frac{b}{2P}+\frac{1}{2}\sqrt{\left(\frac{b}{P}\right)^2-\frac{18\eta v}{\rho g}} \tag{10}$$

After arranging the above equations, the calculation formula of the electricity charged by the oil droplets is [6]

$$q=\frac{4\pi\rho g d}{3U}\left[-\frac{b}{2P}+\frac{1}{2}\sqrt{\left(\frac{b}{P}\right)^2-\frac{18\eta l}{\rho g t}}\right]^3 \tag{11}$$

In many university physics experiment textbooks, the experiment also ignores the influence of air buoyancy, but ignoring the influence of air buoyancy will also bring systematic errors [7]. In order to eliminate this systematic error, air buoyancy is used to derive the calculation formula of electricity [8]. Let the density of the air be ρ', and the equilibrium voltage of the oil droplet when it is stationary in the electric field between the two polar plates of the parallel plate capacitor is U. In order to calculate the oil droplet radius *r*, we still adjust the voltage value between the two polar plates of the parallel plate capacitor to 0. The oil droplets at this time are accelerated to fall under the action of gravity. When the resultant force of the air viscous resistance and air buoyancy on the oil droplets and the gravity on the oil droplets reach a dynamic balance, the oil droplets fall at a uniform speed. The formula for calculating the electricity charged by the oil droplets after finishing is

$$q = \frac{4\pi(\rho-\rho')gd}{3U}\left[-\frac{b}{2P}+\frac{1}{2}\sqrt{\left(\frac{b}{P}\right)^2-\frac{18\eta l}{(\rho-\rho')gt}}\right]^3 \tag{12}$$

From the formula (12), the viscosity coefficient of air can be obtained as

$$\eta = \frac{\rho gt}{18l}\left[\left(\frac{6qU}{\pi(\rho-\rho')gd}\right)^{2/3}+\left(\frac{48qU}{\pi(\rho-\rho')gd}\right)^{1/3}\right] \tag{13}$$

3. Experimental Part

3.1. Instruments and software

In this experiment, the FB809 intelligent Millikan oil drop manufactured by Hangzhou Jingke was used, and EXCEL was used for data processing.

3.2. Data recording and processing

Adjust the three screw knobs on the base plate to make the horizontal bubble centered, and ensure that the upper and lower electrode plates of the oil droplet ionization chamber are in a horizontal state, so that the electric field is parallel to the gravitational field. The selected charged oil droplet is in a static equilibrium state at a certain point in the measurement interface, and the voltage value of "balance U1" is displayed on the upper part of the measurement interface; press the "lift" button to make the "lift" indicator light, and the selected oil droplet rises to the measurement interface The position of the white horizontal line at the uppermost end of the middle; immediately press the "0V" button to make the "0V" indicator light, at this time the voltage between the two electrode plates is 0, the oil droplet moves downward by the action of the gravitational field, and moves to the second line at the upper end of the measurement interface White horizontal line position (when moving at a constant speed), press the "Timer" button to make the "Timer" indicator light on and start timing, when the selected oil droplet drops to the bottom white horizontal line position, press the "Stop" button to make the "Stop" indicator The light is on, the upper end of the measurement interface displays the timer reading, and the lower right corner displays the drop space distance of the oil drop. If the experiment is approved, press the "Confirm/Clear" button to automatically save the U1 and timer readings to the grid on the right side of the screen. The same oil droplet can be measured multiple times (3 to 5 times), or the same measurement can be performed with different oil droplets. After multiple measurements, after the grid on the right is full, press the "Confirm/Clear" button for the last time, then press the "Confirm/Clear" button, and then press the "Query/Return" button, the records of the multiple experimental measurements will be displayed. The experimental data are shown in table 1.

Table 1 Experimental data and processing results

i	U/V	t/s	$q_i(\times10^{-19}C)$	Calculated	Integer	$e_i(\times10^{-19}$ C)	e_i error	$\eta_i(\text{kg}/(\text{m}\cdot s))$	η_i error
1	20	31.831	33.914	21.17	21	1.615	0.8%	1.826	-1.3%
2	56	26.101	15.903	9.93	10	1.590	-0.7%	1.844	-0.3%

3	60	32.248	11.081	6.92	7	1.583	-1.2%	1.904	2.9%
4	53	22.282	22.406	13.99	14	1.600	-0.1%	1.892	2.3%
5	32	20.500	42.275	26.39	26	1.626	1.5%	1.91	3.2%
6	32	23.728	33.630	20.99	21	1.601	0.0%	1.919	3.7%
7	32	23.546	34.039	21.25	21	1.621	1.2%	1.84	-0.5%
8	32	23.899	33.254	20.76	21	1.584	-1.2%	1.84	-0.5%
9	34	30.207	30.692	19.16	19	1.615	0.8%	1.92	3.8%
10	19	20.455	6.287	3.92	4	1.572	-1.9%	1.863	0.7%
11	37	30.132	3.210	2.00	2	1.605	0.2%	1.909	3.2%
12	37	16.638	4.863	3.04	3	1.621	1.2%	1.905	3.0%
13	37	16.278	4.738	2.96	3	1.579	-1.4%	1.917	3.6%
14	37	14.903	6.509	4.06	4	1.627	1.6%	1.911	3.3%
15	19	29.694	3.183	1.99	2	1.592	-0.7%	1.893	2.3%
16	19	26.122	4.872	3.04	3	1.624	1.4%	1.881	1.7%
17	38	36.268	1.573	0.98	1	1.573	-1.8%	1.885	1.9%
18	38	31.961	1.614	1.01	1	1.614	0.7%	1.894	2.4%
19	25	23.159	4.872	3.04	3	1.624	1.4%	1.833	-0.9%
20	25	23.359	4.802	3.00	3	1.601	-0.1%	1.839	-0.6%

Based on the geographical and temperature conditions of the experiment, take the acceleration of gravity $g = 9.79\,\mathrm{m/s^2}$. Air viscosity $\eta = 1.83\times10^{-5}\,\mathrm{kg/(m\cdot s)}$. Atmospheric pressure $p = 81.2\,\mathrm{KPa}$. Oil Density $\rho = 981\mathrm{kg/m^3}$ (Zhonghua 701 Bell Oil). Air density $\rho' = 1.036\,\mathrm{kg/m^3}$. Distance between parallel plates $d = 5\mathrm{mm}$. The distance that the oil droplets fall in space $l = 1.8\mathrm{mm}$. Correction constant $b = 8.22\times10^{-3}\,\mathrm{N/m}$. In Table 1, the serial number *i* represents the number of measurements, *U* represents the measured voltage, and *t* represents the falling time of the oil drop, which is the amount of electricity charged by the oil drop calculated according to the above formula (12). The calculated value represents the basic charge multiple obtained by the reverse derivation method. The integer is the basic charge multiple obtained by the reverse derivation method. e_i is the ratio of q_i to the integer. e_i error is the error between e_i and the theoretical value of the basic charge. η_i is the air viscosity coefficient calculated according to the above formula (13), and η_i error is the error between η_i and the theoretical reference value of the air viscosity coefficient.

It can be seen from Table 1 that the e_i error is within 2%, and the experimental value is relatively close to the theoretical value, which is in line with expectations. The η_i error is within 4%, and the experimental value is relatively close to the theoretical value. It can be seen that this method of measuring the air viscosity coefficient is feasible.

4. Conclusion

In this paper, a method for measuring air viscosity coefficient based on Millikan's oil drop experiment is proposed. Since the radius of the oil droplet is smaller than 10^{-6} meters, the viscosity coefficient of the air is corrected, and the Stokes' law at this time is corrected. In many university physics experiment textbooks, the experiment also ignores the influence of air buoyancy, but ignoring the influence of air buoyancy will also bring systematic errors. In order to eliminate this systematic error, air buoyancy is used to derive the calculation formula of electricity. After formula derivation and experimental measurement, the multiple of the basic charge, the integer value of the multiple of the basic charge, and the basic charge are obtained by the reverse inversion method. This paper further calculates the experimental error value of the basic charge, the calculated value of the air viscosity coefficient, and the experimental error value of the air viscosity coefficient according to the derivation formula. The calculation results show that the error between the air viscosity coefficient measured by this method and the theoretical reference value is within 4%.

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