Empirical Formula for Millimeter-Wave Radar Cross Sectional Area of Naval Vessels

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Abstract: Empirical formula for naval vessels' radar cross sectional area (RCS) has been outdated and inapplicable for millimeter wave. By a simulation method, the model based on the scattering mechanism of radar electric wave was build and verified, then millimeter-wave RCS of naval vessels based on different kinds of condition were imitated at calculator. After a great deal of simulation count outcome were sum up, estimate formulate for millimeter wave RCS of naval vessels was derivative, and then the RCS empirical formula can be used for ka-wave-band.

1. Introduction
The millimeter wave spectrum is between the microwave and the infrared wave, so it has the advantages of both microwave and infrared bands. Compared with microwave, the millimeter wave guidance system has a wide absolute frequency band, narrow antenna wave number, low side lobes, high guidance precision and anti-interference performance; compared with infrared, millimeter wave has strong ability to pass smoke, fog, ash and dust, and has good all-weather combat capability; and the millimeter wave system is small, light in weight, easy to be highly integrated, and it is an ideal frequency band for precision guided weapons. More and more anti-ship missiles use composite guidance or technology with millimeter wave guidance.

The radar cross-sectional area of naval vessels is an important factor affecting the operational capability of anti-ship missiles. The microwave-guided anti-ship missiles are directed to the target by detecting the radar cross-sectional area of the ship. Since the radar scattering characteristics of the target are closely related to the frequency of the radar, the conclusions and applications of the existing radar scattering characteristics of the vessel based on centimeter waves cannot be applied to the millimeter wave. The study of the millimeter-wave radar scattering characteristics of naval ships and the evaluation method of the radar cross-sectional area of naval vessels with millimeter-wave radars have important practical significance for missile attack and defensive operations of naval vessels.

2. Radar Cross-sectional Empirical Formula and Limitations
In the 1970s, the Target Characteristics Department of the US Naval Research Laboratory conducted a large number of actual measurements on the radar cross-sectional area of marine vessels. The measurement uses L, S and X-band microwave radars to perform illumination at a low tilt angle close to the surface of the water, a set of data is measured every 2 degrees according to the target angle of the vessels, for a total of 180 points. After correcting the peak value, the average value is obtained, and the statistical average formula of the cross-sectional area of the maritime target radar is obtained, which is the widely used empirical formula.

\[
\sigma_{\text{average}} = 104 \cdot f^{0.5} \cdot T^{1.5}
\]  

In the formula, \( \sigma_{\text{average}} \) is the total average of the radar cross-sectional area of the vessels, the unit is square meters; \( f \) is the radar frequency, the unit is megahertz; \( T \) is the full displacement of the target ship, the unit is kilotons.
This empirical formula can be used to estimate the radar cross-sectional area of a conventional vessel. However, the limitations of the formula are also obvious; only applicable to the L,S,X and the other centimeter bands, the application for the millimeter band has not to be verified yet, and can not be directly used for the study of the scattering characteristics of naval vessels millimeter wave radar.

3. Simulation Principle of Radar Scattering Characteristics of Naval Vessels

With the rapid development of simulation theory and technology, computer simulation has been widely used in various fields. For the study of the scattering characteristics of the naval vessel’s millimeter-wave radar, the simulation method can be used to simulate the propagation and scattering mechanism of the radar wave, establish a simulation model of the radar work and verify it, and then simulate various conditions on the computer to the millimeter of the naval vessel, the cross-sectional area of the wave radar is simulated and analyzed.

3.1. Radar Scattering Mechanism of Surface Vessels.

The same target presents different radar cross-sectional area characteristics for different radar frequencies. When the wavelength of the incident wave is much smaller than the size of the scattering itself and the scattering centers constituting the scattering, the target radar cross-sectional area is located in the high frequency region. Electromagnetic scattering of naval vessel by millimeter-wave-guided radar is basically high-frequency scattering. Regardless of the other part, in the high frequency region, each part of the target can be considered as independent scattering capability. In this way, the entire vessel can be decomposed into a collection of scattering elements of different shapes, and the scattering field of each unit is calculated according to the shape characteristics and size of each scattering unit, the combination forms the radar cross-sectional area characteristics of the vessel.

High-frequency scattering mainly includes 7 kinds of scattering mechanisms, they are specular scattering, scattering of surface discontinuities (such as edges, corners and apex), scattering of surface derivative discontinuities, scattering of creeping waves or shadow boundaries, traveling wave scattering, scattering of concave areas (such as dihedral angles, trihedral angles) and interactional scattering (such as multi-path superposition or multiple round trips between side-by-side scattering centers). Combining these mechanisms is the total radar cross-sectional area characteristics that can constitute the high-frequency scattering of the composite target.


For a complex target with a high naval vessels radar cross-sectional area, if the hull structure is designed to be fully enclosed, only the specular scattering reflection and surface discontinuity is a strong scattering mechanism, which will significantly affect the radar cross-sectional area characteristics of the vessel. The other five are weak scattering mechanisms that can be ignored. In the treatment of the above two kinds of strong scattering mechanisms, the commonly used methods include geometrical optics, physical optics, and geometrical or physical diffraction theory.

Among them, the geometric optics method starts from ray tracing, and in the extreme case of extreme high frequency, Maxwell’s equation is expressed by optical law. The optical principle defines the properties of the scattering field when the wavelength is sufficiently small that energy flows along the ray path. The task of geometric optics is to determine the path of energy propagation, and the way energy travels from a distant point to an observation point, specifically calculating the amplitude, phase, and polarization of the field.

When applying physical optics, it is assumed that the scattering field contribution to other points on the object from a point is small compared to the incident field strength. Like the integral method of the moment solution, the starting point of the physical optics method is to use the Stratton-Lancheng Zhu scattering field integral equation. When solving the surface inducted current, according to the local principle of the frequency field, completely ignore inductive current between the parts. The mutual influence determines the surface induced current only based on the
independent approximation of the incident field.

Geometric optics and physical optics cannot be used when scattering fields from edges, apex, corners, or shadows must be considered. Geometric or physical diffraction theory must be used to deal with this type of diffraction problem, relying on sharp points. A rigorous solution of a typical structure such as scattering is used to determine its diffraction coefficient, which is expressed as the sum of the physical optical contribution of the surface and the diffraction contribution of the edge, and uses the strict solution of the two-dimensional sharp-point problem to extract the edge contribution.

4. Simulation Modeling and Verification of Radar Cross-sectional Area of Surface Warship

4.1. Simulation Modeling.

Establish a radar cross-sectional area simulation calculation model. According to the high-frequency scattering characteristics of microwave radar and the basic structural characteristics of naval vessels, considering the scattering mechanism of specular reflection and surface discontinuity, geometrical optics, physical optics, geometrical diffraction and physical diffraction theory are used to establish the water naval vessels radar cross-sectional area simulation model.

Establish a three-dimensional model of naval vessels. Using Creator 3D production software, a three-dimensional model of two types of conventional naval vessel (type I and type II) was established as simulated vessel model input. Among them, the standard displacement of type I naval vessels is 2700 tons, and the standard displacement of type II naval vessel is 7000 tons.

4.2. Simulation Parameter Setting.

Microwave Propagation Parameters. According to the structural characteristics of the naval vessels, the scattering characteristics parameters of the microwave in the hull are set, including the number of bounces between the various parts of the vessel before the radio wave returns to the receiver, and the angle range of the apex of corner when the diffraction calculation is allowed.

Radar Operating Parameters. According to the performance characteristics of the anti-ship missile terminal guidance radar, the radar operating parameters are set, including radar carrier frequency, polarization mode, and different radio wave incident angle azimuth, elevation angle to calculate the target radar cross-sectional area value, amplitude and phase value.

Other Calculation Parameters. The incident azimuth is set to be in the range of 0-359 degrees and 1 degree interval. Since the anti-ship missiles mostly fly in parallel to the sea, the radio wave pitch angle is set to zero degrees in the simulation calculation, which is also in accordance with the test conditions of “low elevation angle” in the empirical formula. The obtained simulation results are easy to compare with empirical formulas.

4.3. Simulation Data Processing.

Since the radar electromagnetic waves reach the target and generate secondary scattering, the scattered waves reinforce each other in some phases, causing the radar cross-sectional area to “suddenly out” in certain specific directions. These outgoing points do not reflect the overall characteristics of the radar cross-sectional area.

\[
\begin{align*}
\sigma_{\text{average}} &= \left(\sum_{i=1}^{n} 10^{\frac{\text{RCS}_i}{10}} \right) / n \\
\text{RCS}_{\text{average}} &= 10 \times \log(\sigma_{\text{average}})
\end{align*}
\]

In the formula, \(\sigma_{\text{average}}\) is the total average value of the radar cross-sectional area, the unit is square meter; \(\text{RCS}_i\) is the radar cross-sectional area value of the first azimuth of the simulation output, the unit is decibel square meter; \(n\) is the number of input radio wave incident azimuths; \(\text{RCS}_{\text{average}}\) is the average value of the radar cross-sectional area after conversion of \(\sigma_{\text{average}}\), the unit is decibel square meter.

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4.4. Simulation Model Verification.

The empirical formula and simulation algorithm are used to calculate and compare the cross-sectional area of the vessel's centimeter wave and millimeter wave radar to verify the validity of the simulation model and extrapolation availability of the empirical formula.

The centimeter wave takes the L, S and X bands, and its center frequency is typically 1.3, 2.8, 9.23GHz. Since the atmospheric propagation attenuation of millimeter wave is very serious, the terminal guidance radar should try to work in the “atmospheric window”. The typical band is the ka band, and the center frequency is typically 35GHz.

The simulation results of the radar cross-sectional area of the I-type naval vessel are shown in Fig. 1.

![Fig 1. RCS simulation of Ship I](image1)

In the figure, the abscissa is the azimuth angle of the electric wave incident, and the ordinate is the cross-sectional area of the radar of the vessel. Rotating the vessel right and horizontal as a reference for one circle, the radar cross-sectional area curve is obtained, and the horizontal line is the arithmetic mean $\sigma_{\text{simulation}}$ of the simulated value of the radar cross-sectional area. The comparison result of $\sigma_{\text{simulation}}$ in the empirical formula to calculate the value $\sigma_{\text{experience}}$ is shown in Fig 2 and Fig 3. By comparison, it’s not difficult to find:

The calculated values of the centimeter band (L, S, X) are not much different. Since the empirical formula is a statistical result obtained by performing a large number of actual measurements on the target of the sea project, the calculation comparison results show that the simulation is basically consistent with the actual measurement. Therefore, the simulation model can be considered to be effective and can be used for the study of the radar cross-sectional area of the naval ships.

The millimeter band (ka) has a large difference between the two calculated values. Since the simulation is modeled based on the radar scattering mechanism of naval vessels, it is applicable to any radar band and it is empirically validated. Calculating the comparison results of the surface empirical formula has a large calculation error in the ka band. Therefore, it can be considered that the empirical formula cannot be directly extrapolated. The simulation method is an effective way to obtain the radar cross-sectional area data of the naval vessel millimeter wave.

![Fig 2. RCS comparisons of Ship I](image2)  ![Fig 3. RCS comparisons of Ship II](image3)

5. Estimation Formula of Radar Cross-sectional Area of Surface Ship Millimeter Wave

If we draw on the method of dealing with the empirical formula of the radar cross-sectional area of the naval research by the target characteristics department of the US Naval Research Laboratory, a large number of simulation calculation results are summarized and the estimation formula of the
radar cross-sectional area of the naval vessel millimeter wave is derived. The application range and flexibility of the simulation method can be greatly improved.

It can be seen from the simulation that the shorter the radar wavelength, the larger the radar cross-sectional area of the vessel. And its degree of reinforcement increases as the tonnage of the vessel increase, there is a complex nonlinear relationship between the radar cross-sectional area of the ship and the radar carrier frequency f and the target tonnage T. Taking the ka-band as a breakthrough, the cross-sectional area of the ka-band radar of the naval vessel is compared with the radar cross-sectional area of the L, S and X-band, if “kj” is used to represent the ratio of the two, then the value of “kj” should be a decreasing function of f, an increasing function of T. The other types of conventional surface ship models are built for simulation calculation. The fitting relationship between “kj” and f, T is as follows:

\[ K_j = \left(\frac{f_{ka}}{f}\right)^{0.5} \times T^{0.7} \]  

If “kj” is used as the “millimeter wave correction coefficient” and multiplied by the original radar cross-sectional area empirical formula. the radar cross-sectional area estimation formula of the surface ship ka-band can be obtained:

\[ \sigma_{ka} = 104 \times f_{ka}^{0.5} \times T^{2.2} \]  

The estimation formula is based on the traditional empirical formula of radar cross-sectional area, and is corrected according to the statistical results of a large number of simulation calculations. The extrapolation of the empirical formula of the radar cross-sectional area in the ka-band is realized. The radar cross-sectional area of the ka-band of conventional tonnage vessels of different tonnages are shown in Fig.5, and some of the calculated values have been verified by the sea test.

6. Conclusion

The simulation method is an effective way to obtain the radar cross-sectional area data of the naval vessel millimeter wave. The calculation formula of the radar cross-sectional area of the ka-band of the naval vessel obtained by the simulation method has strong maneuverability. It should be pointed out that in order to further improve the practical application value of simulation methods and estimation formulas, a lot of research work is needed, including higher-precision naval vessel model, sea clutter model, electric wave conduction model of vessel stealth coating and development, as well as conducting offshore surveys on the radar cross-sectional area of millimeter-wave, etc.

References