The Application of Three Scale Fall and Rise Temperament and Twelve-tone Equal Temperament in Violin Performance

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Keywords: Twelve-tone equal temperament; Violin performance; Temperament modulation

Abstract: We have been trying to sum up a more scientific method in the practice of playing and teaching to add bricks to it. However, it is regrettable that the development speed of violin playing theory does not match the rhythm of the times. This article proposes apply the three-scale fall and rise temperament and twelve-tone equal temperament in violin performance, and provides a reference for the violin performance theory to find new growth points.

1. Introduction

The violin is a melody instrument, and it can also play harmony and polyphonic work. Specifically, the violin can play single-melody music, as well as two-tone, three-tone chords, and four-tone chords [1]. Relevant research at home and abroad has confirmed that the use of temperament in violin performance is a complicated acoustic phenomenon. The complexity is that the temperament used by the violin is not a simple temperament, but a variety of temperaments. The research on which temperament modulation should be used for violin performance has always been a hotspot in the field of violin music and art.

2. The problem of using the violin system for violin performance

The question of violin playing intonation has always been controversial. Of course, most people think that since the violin is a single-tone melody-based instrument, plus the fifth-degree fixed string and other factors, the violin should be played with a fifth degree resonance [2]. In recent years, it has been suggested that pure temperament should be used as the standard for violin pitch [3].

However, due to the different pitches of different scales, the pitch of the violin is not fixed. A tone has different positions in different temperaments. Although this change in position is subtle, it actually exists, which requires the player to make corresponding adjustments in the performance to adapt to the different requirements of different pitches [4]. It is the broadness and variability of the violin's use of the temperament that makes the violin intonation very complicated and difficult to grasp.

According to the auditory habits, the requirements for the pitch of single-part music and multi-part music are different. When listening to monophonic music, appreciators hope that the semitones are closer, so that the music tends to be stronger. When listening to multi-voice music, in addition to the horizontal pitch of the music, it is also required to achieve the most harmonious degree when different voices are combined at the same time. To achieve these goals, any single temperament is indispensable. This article explores the combination of the twelve-tone equal temperament and three scale fall and rise temperament, and proposes to apply these two temperament systems in violin performance to meet the hearing the aesthetic requirements provide the basis.
3. Twelve-tone equal temperament combined with three scale fall and rise temperament

3.1. Twelve-tone equal temperament

The twelve-tone equal temperament is a temperament system that divides an octave into twelve semitones with equal frequency ratio. So, it is also called “twelve equal ratio” (referred to as “equal ratio”). Take C major as an example.

The twelve semitones of the twelve-tone equal temperament are all equal 100 cent, major second: 200 cent, major third: 400 cent, minor third: 300 cent, pure fourth: 500 cent, pure fifth: 700 cent, major sixth: 900 cent, minor Seven degrees: 1000 cent, major seventh: 1100 cent, pure octave: 1200 cent. The twelve semitones are all uniform 100 cent. Because the twelve-tone equal temperament has a balanced beauty, the twelve-tone equal temperament is generally used in the transposition.

3.2. Three scale fall and rise temperament

The three-point profit and loss method is the world's first well-documented mathematical calculation method. When the temperaments obtained by this method form a system, they form a three scale fall and rise temperament. Now we use $S_n$ to represent the twelve notes in a scale of the temperament. It is formed by multiplying two numbers $p = 3/2, q = 3/4$. We take the lead sound $S_1 = 1$, and then set up a set of data, $1, p, pq, (pq)^2, (pq)^3, (pq)^4, (pq)^5, (pq)^6$. Its result can be represented by a line graph, and below the graph (figure 1).

As we can see in Figure 1, $p$ is greater than 1, and the increase from $S_1$ to $S_8$ is $3/2$. $q$ is less than 1, so $q/p = 98$ drops from $S_8$ to $S_3$. After $p^3q^3$, the order is reversed, decreasing first and then increasing. The disadvantage of the thirds temperament of fall and rise is that $(pq)^6$ is not equal to 2, although we still use C to represent this sound, in fact it is not the full octave of $S_1$, that is, the so-called “cannot be turned into a palace”.

3.3. Three scale fall and rise temperament combined with twelve-tone equal temperament

Although the three scale fall and rise temperament in ancient China has the above-mentioned shortcomings, it occupies an important position in the history of music technology. It is closely related to the tuning technique of modern keyboard instruments. In order to preserve the status that this method should have in the history of world music, we made a broader improvement based on
its original theory, making it related to the twelve-tone equal temperament, and established a theoretical basis for the tuning procedure.

To make the ratio of the octave to the main tone 2 to 1, we change two numbers \( p' \) and \( q' \), which are equivalent to \( p \) and \( q \) in the old system. These two new numbers must meet the following conditions:

\[
(p'q')^6 = 2
\]

In order to preserve the pure pitch of the G sound, we make \( p' \) the same as the original \( p \), namely

\[
p' = \frac{3}{2}
\]

Substitute \( p' \) into (1) to find

\[
q' = \frac{3}{2^{1/6}} = 0.7483...
\]

It is slightly smaller than \( q \) (= 0.75) in the three scale fall and rise temperament. From this number, we still follow the steps of formula (3) to find each sound. This improved three scale fall and rise temperament will be represented by \( T \), and their values are listed as follows:

<table>
<thead>
<tr>
<th>( T )</th>
<th>C</th>
<th>C#</th>
<th>D</th>
<th>D#</th>
<th>E</th>
<th>F</th>
<th>F#</th>
<th>G</th>
<th>G#</th>
<th>A</th>
<th>A#</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Compared with the twelve-tone equal temperament, all odd-numbered sounds \( T_1, T_3, T_5, T_7, T_9, T_{11}, T_{13} \) are identical to \( R_1, R_3, R_5, R_7, R_9, R_{11}, R_{13} \). The difference between the other six-notes and the six-notes in the average temperament is less than two-notes. The so-called cent is to explain its definition, assuming that the frequency ratio of the two tones is \( f_m / f_n \), then the difference between the two tone is represented by \( C_{mn} \), which is

\[
f_m \quad f_n = \frac{C_{mn}}{1200}
\]

For example, the difference between \( f_m \) and \( f_n \) is half-tone, that is

\[
f_m \quad f_n = 2^{1/7} = 2^{1/1200} = 100
\]

Then,

\[
C_{mn} = 100
\]

Conversely, if the two sounds differ by 2 cents, the ratio of these two sounds is \( f_m / f_n = 2^{100} = 1.001155... \)

The musical rhythm created by \( p' \), \( q' \) can be called \( p' \), \( q' \) upper and lower coherence temperament, which is more accurate. This temperament has the advantage that it preserves two just intonation of G (= 3/2) and F (= 4/3) in C major. The G sound is a pure fifth, which occupies an important position in ancient Chinese and Greek music. These are the two basic numbers in the old three scale fall and rise temperament, because \( p = 3/2 \) and \( q \) is the reciprocal of 4/3. We have a detailed analysis in the next section.

3.4. Pure temperament

Pure temperament is a very controlled rhythm in the history of music. Its scale can be created by three data or intervals. These three intervals are

\[
M = \text{Daquan} = \frac{9}{8}
\]

\[
m = \text{Small tones} = \frac{10}{9}
\]

\[
s = \text{Semitone} = \frac{16}{15}
\]
In a major key, such as C major, its scale is generated using the program in the following list:

<table>
<thead>
<tr>
<th>Musical alphabet</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formula</td>
<td>1</td>
<td>M</td>
<td>Mm</td>
<td>Mms</td>
<td>M’s</td>
<td>M’ms</td>
<td>M’s</td>
<td>M’ms</td>
</tr>
<tr>
<td>Numerical value</td>
<td>9</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>15</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

All normalized frequencies in pure temperament are simple fractions. Among them, F and G two sounds are the same as \((p', q')\) in the coherent rhythm. \(T_6\) and \(T_8\) two sounds are the same, and its octave is also twice as high as the main sound. In the reference book, the author calculated the frequencies of 15 major and 15 minor keys in the pure temperament. We calibrated all the data and found only one error: the Eb 4 tone in Bb minor should be 316.6Hz, not 313.2Hz, which is probably a printing error \(^5\). Among the 30 major and minor keys, the pure temperament has 36 different tones, so the pure temperament should not be used for the transposition of the symphony orchestra, but it still has its advantages when playing in the chamber string or in the chorus without accompaniment. The temperament is in harmony with harmony. Like its major third harmony:

- C, E, G;
- F, A, C;
- G, B, D;

They all fit in the 4: 5: 6 ratio, and the simple ratio seems to be pleasant, nice, and in the harmony of the minor third.
- A, C, E;
- D, F, A;
- E, G, B;

They all fit in the 10:12:15 ratio and they are coordinated.

In the harmony above, C is the octave of C, and A is the lower octave of A. There is no such pure harmony in average temperament. There are other sounds in the pure temperament. There are many introductions in the reference. Among them, there are two sounds mentioned, which are called:

- Four degrees = \(\frac{45}{32}\) = 1.40625,
- Increase by five degrees = \(\frac{64}{45}\) = 1.4222…

The geometric mean of these two tones is the square root of \(2\left(\frac{45}{32} \times \frac{64}{45}\right)^{\frac{1}{2}} = 2^{\frac{1}{2}}\)

We think that the increase of four degrees can be represented by 7/5, and the decrease of five degrees can be represented by 170. These two numbers are very different from (15), (16), because 7/5 = 1.4 (10) 107 = 1.42857… (11). The geometric mean of these two tones is also the square of 2, but these two numbers are simpler than the former. The various intervals of pure temperament are shown in Figure 2.
4. Conclusion

The three scale fall and rise temperament and the twelve-tone equal temperament are applied to the violin performance, which can not only perform fast scales and slow scales, but also perform chords and chromatic scales better, and realize the complementary advantages of the twelve-tone equal temperament and the five-degree mutualism temperament.

References