# Research on the Synergistic Application of Least Squares Optimization and the Hungarian Algorithm in the Localization of Rocket Debris

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**Abstract:** This study aims to accurately pinpoint the landing point of rocket debris by establishing mathematical models and applying optimization algorithms. Initially, we constructed a model to determine the positional coordinates of rocket debris at the moment of the sonic boom. By analyzing the collected data from various monitoring devices using multi-source localization methods, we concluded using our established model. During the model construction, we transformed the Earth's geographical coordinates into a Cartesian coordinate system, simplifying the distance calculations. We employed the least squares method and gradient optimization algorithms to solve the equations, thereby minimizing errors and ultimately determining the positional coordinates and the time of the sonic boom of the rocket debris. Although the results of the gradient descent algorithm were almost unusable, we adopted a method based on nonlinear least squares. Furthermore, we explored the problem of multi-debris sound source identification and localization, utilizing the Hungarian algorithm for matching, defining a cost matrix function, and optimizing with the linear assignment function. By validating the model with data recorded from monitoring devices, we analyzed the sound source localization of rocket debris. Ultimately, we proposed a single-point localization model for rocket debris, based on multi-source localization and least squares optimization, solving the problem of landing points for multiple debris in a short period. This study not only provides a new method for the localization of rocket debris but also lays the foundation for future related research.

## 1. Introduction

In the modern space industry, predicting and accurately locating the landing sites of rocket debris after launch is crucial for ensuring ground safety, environmental protection, and resource recycling [1]. With the increasing frequency of rocket launches, the rapid and precise location of rocket debris has become an urgent issue to be addressed in the field of space safety. The sonic boom generated by rocket debris when re-entering the atmosphere provides a unique signal source. By analyzing these sound wave signals, the landing position of the debris can be inferred. Although previous studies have made some progress in the localization of rocket debris, most methods still have limitations in dealing with complex environmental factors and the integration of multi-source data [2]. Moreover, with the development of rocket technology, debris landing prediction and positioning face more complex challenges, such as high dynamics, multi-source interference, and data uncertainty [3]. Therefore, developing a new and efficient method for rocket debris localization is of significant practical importance for improving positioning accuracy and response speed.

This study focuses on the analysis and processing of the sonic boom signals of rocket debris, proposing a synergistic application method based on least squares optimization and the Hungarian algorithm [4, 5]. By constructing mathematical models, we transform the Earth's geographic coordinates into a Cartesian coordinate system to simplify the calculation process. At the same time,



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we use multi-source localization technology, combined with the least squares method and gradient optimization algorithms, to accurately calculate the position of the rocket debris [6]. Additionally, for the situation where multiple debris land simultaneously, we introduce the Hungarian algorithm to achieve optimal source matching and localization. The innovation of this study lies in the fact that we not only propose a new method for rocket debris localization but also demonstrate the effectiveness and feasibility of this method in dealing with complex environmental rocket debris localization issues through the validation of actual monitoring data.

## 2. Single Point Localization of Rocket Debris

Firstly, we established a model to determine the position coordinates of a single rocket debris at the time of its sonic boom. Utilizing the multi-lateration method, we analyzed data collected from various monitoring devices and, through our established model, reached a conclusion.

### 2.1. Model Construction

We transform the Earth's geographic coordinates into a Cartesian coordinate system using an idealized model, which assumes that the distance between adjacent degrees of latitude and longitude is constant. This simplification facilitates the calculation of distances. The target equations we have formulated are as follows:

$$\mathrm{v}(t_i - t_0) = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} \tag{1}$$

Where, x, y, z represent the position coordinates, $t_0$ denotes the wave propagation time, the speed of sound v is 340 m/s, and the subscript i refers to the data from the i-th device.

In the application of the aforementioned equations, we have utilized optimization algorithms such as the least squares method and gradient optimization to solve the equations, thereby minimizing the errors. Ultimately, we can determine the position coordinates of the rocket debris and the time of the sonic boom. The calculation method described above is a simplified algorithm that assumes a Cartesian coordinate system and disregards the impact of environmental factors on the location. We will now provide a detailed description of the computational methods used in the process: multi-lateration, gradient descent, and nonlinear least squares method.

### 2.2. Muti-lateration Method

The information derived from the time-distance relationship is radial information, which allows us to determine the specific location of the sonic boom using the positional information of multiple sensors and the data they monitor. First, for each sensor, we establish the equation as shown in (1). Subsequently, we define the objective function as the sum of the squares of the differences between the times measured by each sensor and the data obtained from the multi-lateration method. The function is as follows:

$$\sum_{i=1}^{n}(v \times (t - t_0) - \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2})^2 \tag{2}$$

With the aforementioned function serving as our objective function, we tackle the optimization problem. We explore this through two distinct approaches: the least squares method and the gradient descent method.

#### 2.2.1. Nonlinear Least Squares Method

Based on Equation (1), we can deduce that:

$$t_i = t_0 + \frac{\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2}}{v} \tag{3}$$

In order to reduce the error between the measured time data from the monitoring devices and the theoretical time data, we establish the following function based on the aforementioned formula:

$$\mathrm{f(x, y, z, t_0)} = \sum_{\mathrm{i=1}}^{\mathrm{n}}[\mathrm{t} - (\mathrm{t_0} + \frac{\sqrt{(\mathrm{x-x_i})^2+(\mathrm{y-y_i})^2+(\mathrm{z-z_i})^2}}{\mathrm{v}}]^2 \tag{4}$$

We collected 7 sets of data recorded by monitors used for locating the position of rocket debris, which include the times at which the sonic booms from the debris were detected by different monitors. To accurately calculate the positional coordinates, at least four sets of data are required. This is because, in three-dimensional space, data from at least four non-coplanar monitoring points are needed to determine the exact position of a point. Therefore, we randomly selected four sets out of the seven, which can generate a variety of different data combinations, each of which can be used to independently perform a position calculation. We utilized the nonlinear least squares method to find the parameters that minimize the objective function, with the results shown in Figure 1.

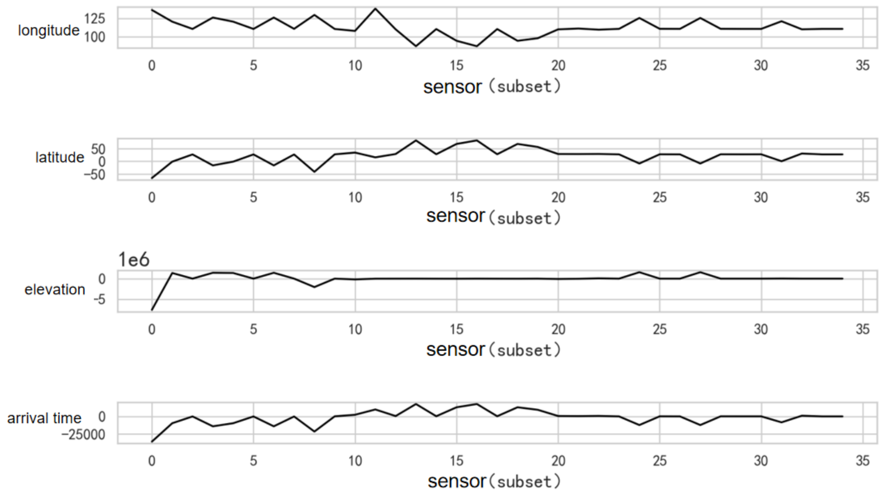


Figure 1 Data Visualization

In the figure above, the four rows represent longitude, latitude, elevation, and sonic boom arrival time, respectively, with 35 sub-arrays horizontally. It can be observed that although the sonic boom arrival time for sub-array 0 is impractical, the rest are reasonable, demonstrating that the method is feasible to a considerable extent. We conducted an analysis of different initial parameters to examine their impact on the results. We analyzed all 35 outcomes generated from the combinations of the 7 sets of data, and the analysis results are shown in Figures 2 and Figure 3.

From the linear graphs presented, it is evident that although the results vary with different initial conditions, the range of variation is still within an acceptable range. It is not necessary to select specific conditions to obtain the correct results. Therefore, it is concluded that the multi-lateration method for rocket debris based on the least squares method is reasonable and usable.

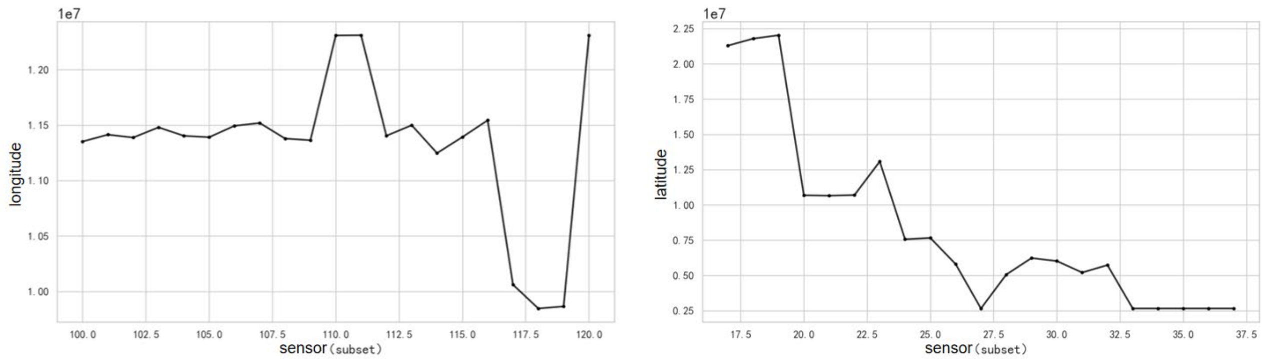


Figure 2 The Impact of Different Longitudes and Latitudes

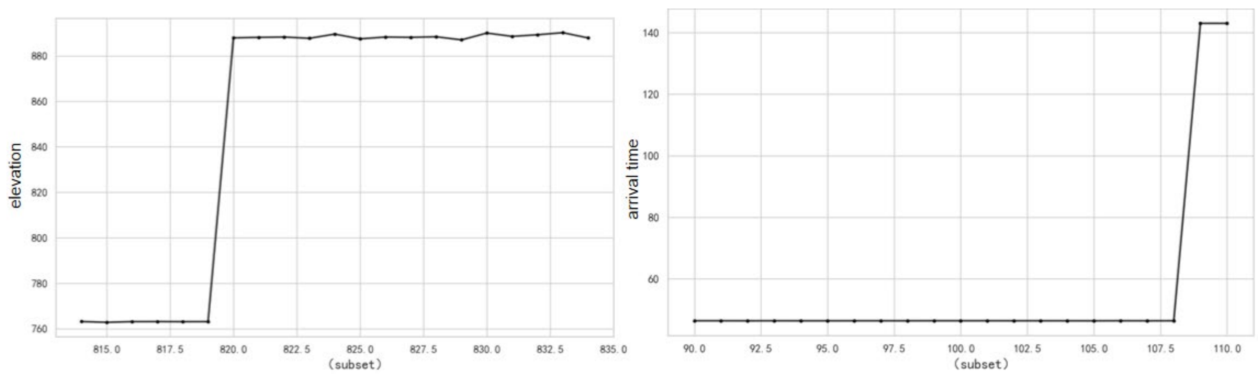


Figure 3 The Impact of Different Elevations and Times

### 2.2.2. Gradient Descent Algorithm

Gradient Descent is a commonly used optimization algorithm widely applied in machine learning, deep learning, and various numerical computations. Its main function is to iteratively adjust the model parameters to minimize the value of the objective function, thereby optimizing the model. The basic idea of gradient descent is to update the parameters in the direction of the steepest descent of the objective function, which is the negative gradient direction, to achieve a local or global minimum. The objective function is usually a loss or cost function, and the gradient of this function with respect to the parameters is computed and then used to update the parameters.

The gradient is the partial derivative of the function at a specific point, representing the rate of change of the function at that point. For a multi-dimensional parameter vector, the gradient is a vector composed of the partial derivatives of each parameter, indicating the direction and speed of the function's change in each dimension.

Through our calculations, we found that the results obtained from the gradient descent algorithm were nearly unusable. Therefore, in subsequent research, we adopted a method based on the nonlinear least squares approach.

## 3. Multi-Debris Sonic Boom Source Identification and Localization

### 3.1. Model Construction

First, we state an assumption: Technical personnel analyze the landing positions of each debris by integrating various data and environmental factors at the time of detachment. Based on these positions, the Hungarian algorithm is used to perform the matching algorithm. The cost matrix function is defined as follows:

$$A_{ik} = \left|t_{ik} - T_{ij}\right| \quad (5)$$

$$T_{ij} = T_{0j} + \frac{d_{ij}}{v} \quad (6)$$

Where, $t_{ik}$ represents the actual time at which the i-th sensor receives the k-th group of signals. $T_{ij}$ represents the theoretical time for the j-th rocket debris to reach the i-th sensor. $d_{ij}$ represents the distance from the j-th rocket debris to the i-th sensor. $A_{ik}$ represents the i-th sensor's reception of the k-th group of signals (cost matrix element). v represents the speed of sound. $J_{ik}$ represents the decision variable for the i-th sensor's k-th group of signals.

The linear assignment function is given by:

$$\min \sum_{i=1}^{n} \sum_{k=1}^{m} A_{ik} J_{ik} \quad (7)$$

In which the values of $J_{ik}$ are either 0 or 1, with 1 representing that the i-th sensor receives the k-th group of signals from the j-th rocket debris, and 0 indicating that it does not come from that particular rocket debris.

### 3.2. Hungarian algorithm

The Hungarian algorithm is an optimization method for solving multi-task assignment problems, particularly suitable for problems that can be abstracted as complete bipartite graphs, such as the maximum matching problem in bipartite graphs, which is essentially equivalent to solving task assignment or allocation problems. For example, when there are n tasks to be assigned to n individuals, the Hungarian algorithm aims to determine the best task allocation plan to ensure the highest overall efficiency. The specific steps include: first, simply process the original matrix by subtracting the smallest element of each row and column to form a new matrix; then, mark the zero elements and try to cover all zero elements with the fewest number of lines, if the number of lines required equals the order of the matrix, then the optimal solution is found. If the number of lines is insufficient, further processing of the processed matrix is required: subtract the value from the row where the smallest value outside the zero element is located, and add the value to the column where it is located, then re-mark the zero elements. Through such iterative processing, the Hungarian algorithm ensures that each task is assigned at the minimum cost, thereby achieving the optimal allocation between tasks and assignees.

### 3.3. Model Validation and Optimization Strategies

Based on the data from the monitoring devices, which recorded the arrival times of the sonic booms from the rocket debris, along with the positional coordinates of the monitoring devices, including longitude, latitude, and elevation, we utilized the aforementioned models to pinpoint the exact locations and times of the sonic booms for four pieces of rocket debris. The data is presented in Table 1.

Table 1 Research Data

| Equipment | Longitude | Latitude | Elevation | Sonic Boom Arrival Time |
|---|---|---|---|---|
| A | 110.241 | 27.204 | 824 | 100.767 |
| B | 110.783 | 27.456 | 727 | 92.453 |
| C | 110.762 | 27.785 | 742 | 75.560 |
| D | 110.251 | 28.025 | 850 | 94.635 |
| E | 110.524 | 27.617 | 786 | 78.600 |
| F | 110.467 | 28.081 | 678 | 67.274 |
| G | 110.047 | 27.521 | 575 | 103.738 |

We perform random initialization for prediction, and the results are shown in Figure 4.

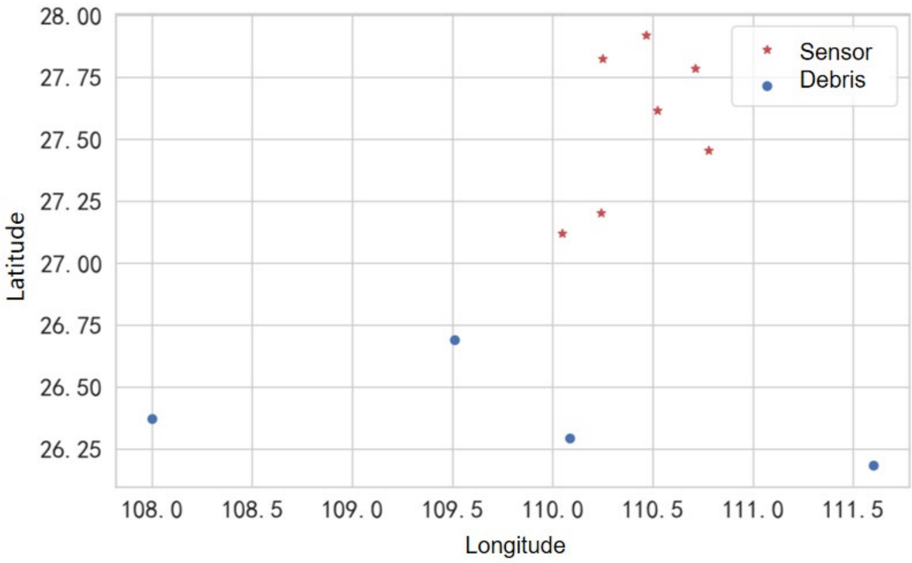


Figure 4 Preliminary Prediction Results

It can be observed that different monitoring points receive seismic waves from different debris in varying orders, hence an optimization algorithm can be utilized for allocation, with the results of the allocation shown in Figure 5.

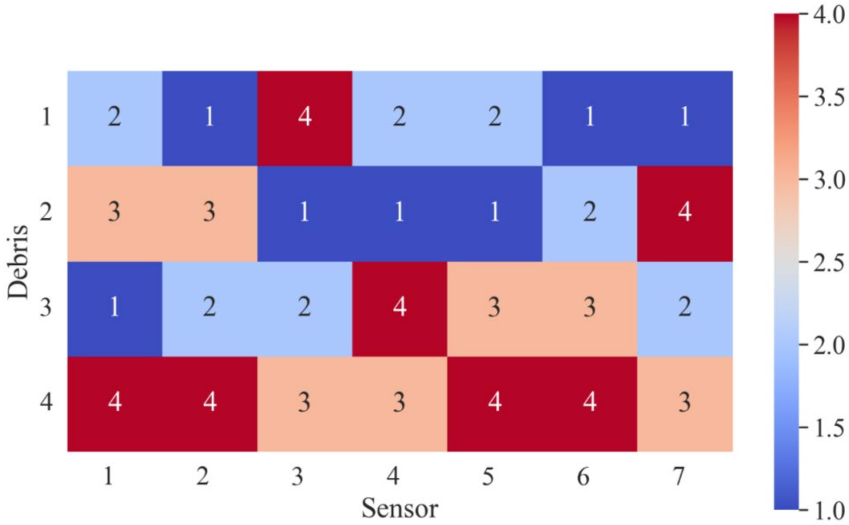


Figure 5 Optimization algorithm allocation results

Ultimately, we will calculate the theoretical time obtained from distance calculations and subtract the actual time to construct a cost matrix, implementing the optimal allocation. By treating the problem of multiple debris falling within a short time as individual problems, we then use a rocket debris single-point positioning model based on multi-lateration and least squares optimization to solve for the specific location coordinates of each piece of debris.

## 4. Conclusion

The study presents a robust model for rocket debris localization, leveraging the least squares optimization and the Hungarian algorithm to accurately pinpoint the landing points based on sonic boom signals. The model's performance was found to be reliable across different initial conditions, with the Cartesian coordinate transformation streamlining the computational process. Despite the initial challenges with the gradient descent algorithm, the adoption of a nonlinear least squares approach proved more effective.

While the model has shown promising results, future work will focus on refining the algorithmic components and incorporating real-time environmental data to enhance its predictive capabilities. Additionally, exploring advanced machine learning techniques could further improve the model's adaptability to dynamic conditions. The ultimate goal is to develop a system that not only provides precise localization but also operates in real-time, ensuring swift and effective response to rocket debris landings.

## References


[1] Kawamoto Y, Nishiyama H, Kato N, et al. A traffic distribution technique to minimize packet delivery delay in multilayered satellite networks[J]. IEEE Transactions on Vehicular Technology, 2013, 62(7): 3315-3324.

[2] Karacalioglu A G, Stupl J. The impact of new trends in satellite launches on the orbital debris environment[C]//IAASS Conference aEurooeSafety First, Safety for AllaEuro. 2016 (ARC-E-DAA-TN31699).

[3] Liever P A, Gale M P, Mehta R S, et al. Gas-granular simulation framework for spacecraft landing plume-surface interaction and debris transport analysis[C]//16th Biennial International Conference on Engineering, Science, Construction, and Operations in Challenging Environments. Reston, VA: American Society of Civil Engineers, 2018: 39-48.

[4] Morrison D D. Optimization by least squares[J]. SIAM Journal on Numerical Analysis, 1968,

5(1): 83-88.

[5] Hamuda E, Mc Ginley B, Glavin M, et al. Improved image processing-based crop detection using Kalman filtering and the Hungarian algorithm[J]. Computers and electronics in agriculture, 2018, 148: 37-44.

[6] Ahmadianfar I, Bozorg-Haddad O, Chu X. Gradient-based optimizer: A new metaheuristic optimization algorithm[J]. Information Sciences, 2020, 540: 131-159.