Research on Sonic Boom Localization of Rocket Debris Based on BFGS and Genetic Algorithms

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Abstract: In order to accurately determine the position and time of a sonic boom generated by rocket debris, this study initially converts the geographical coordinates of observation equipment into a Cartesian coordinate system for mathematical modeling. Subsequently, a multi-lateration technique is utilized to construct an optimization model, aiming to minimize the sum of squared differences between the predicted and actual observed arrival times of the sonic boom. To address this optimization challenge, the paper employs the BFGS algorithm, an effective numerical optimization method capable of identifying parameter values that minimize the objective function. Through iterative computation, the algorithm predicts the three-dimensional coordinates and specific timing of the sonic boom occurrence. Finally, the study demonstrates the accuracy of the model's predictions through three-dimensional visualization results, laying the groundwork for subsequent processing of more complex multi-debris localization issues.

1. Introduction

The debris generated during rocket launches and reentries may produce sonic booms as they traverse the atmosphere [1]. Accurately localizing these sonic booms is crucial for understanding rocket performance, assessing potential damage risks, and facilitating effective debris recovery. Traditional methods for sonic boom localization rely on ground or aerial observation equipment, calculating the source's position based on the time differences of the boom's arrival. However, the propagation speed of sound waves in the atmosphere is influenced by various factors, including temperature, humidity, and wind speed, making precise localization of sonic booms a complex issue [2].

To enhance localization accuracy, this study proposes a method for localizing sonic booms from rocket debris based on the BFGS algorithm and genetic algorithms [3, 4]. The BFGS algorithm, or Broyden-Fletcher-Goldfarb-Shanno algorithm, is a widely used quasi-Newton method that optimizes the objective function by iteratively updating an approximate inverse Hessian matrix, particularly suitable for nonlinear problems in high-dimensional spaces. Genetic algorithms, mimicking natural selection and genetic mechanisms, optimize solutions over multiple generations through selection, crossover, and mutation operations, making them suitable for complex multi-variable optimization problems.

This paper begins by introducing the fundamental theory of sonic boom localization and then elaborates on how to convert the geographical coordinates of observation equipment into a Cartesian coordinate system [5], constructing a multi-lateration optimization model. Subsequently, the BFGS algorithm is employed to solve the model, predicting the three-dimensional coordinates and occurrence time of the sonic boom [6]. Additionally, this study explores the synchronous localization of multi-source sonic boom signals and proposes a multi-debris localization model based on genetic algorithms. Finally, the accuracy of the model is validated through three-

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dimensional visualization results, and its application prospects in rocket debris recovery and utilization are discussed.

2. Fundamentals of Sonic Boom Localization for Rocket Debris

In order to accurately pinpoint the position and time of a sonic boom from rocket debris through data recorded by observation equipment, this study initially involves converting the geographical coordinates of the observation devices into a Cartesian coordinate system for mathematical modeling. Subsequently, a multi-lateration technique is utilized to construct an optimization model, aiming to minimize the sum of squared differences between the predicted and actual observed arrival times of the sonic boom. To solve this optimization problem, the paper employs the BFGS algorithm, an effective numerical optimization method that can find the parameter values that minimize the objective function. Through iterative computation, the algorithm can predict the three-dimensional coordinates and specific time of the sonic boom occurrence. Finally, the study demonstrates and validates the accuracy of the model's predictions through three-dimensional visualization results, laying the groundwork for subsequent processing of more complex multi-debris localization problems.

2.1. Data Analysis

To present the initial positions of the observation equipment more intuitively, we have visualized the initial data as shown in Figure 1. The initial data is shown in Table 1.

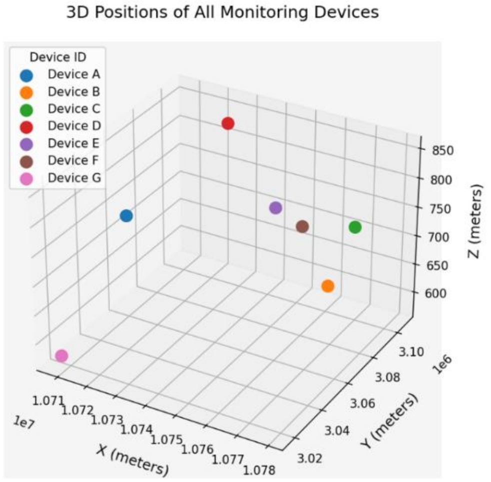

Figure 1 Initial data visualization results

Table 1 Initial data

Equipment	Longitude	Latitude	Elevation	Sonic Boom Arrival Time
A	110.241	27.204	824	100.767
B	110.783	27.456	727	112.220
C	110.762	27.785	742	188.020
D	110.251	27.825	850	258.985
E	110.524	27.617	786	118.443
F	110.467	27.921	678	266.871
G	110.047	27.121	575	163.024

In this Cartesian coordinate system, it can be intuitively seen that the observation device A is in a higher position in the figure, while the observation device G is in a lower position. As can be seen from the three-dimensional coordinate diagram, this distribution indicates that each observation device is arranged in a geographical position with different Z-axis (height). Each observation device can intuitively detect and lock the sonic boom signal data from different directions and different latitudes in the space.

The three-dimensional coordinate result also clearly shows the relative distance between each observation device, which is of great importance for subsequent calculation and analysis. For example, the time difference of the arrival of sound waves is used to locate the three-dimensional coordinate position of the sound burst, and the arrival time of the sonic boom is used to increase the three-dimensional visualization of the sound wave propagation sphere. A sphere that converts the sonic boom arrival time into a radius (taking into account the speed of sound, depending on the situation). In order to clearly show the spheres of the sonic boom observed by the seven observing devices, each sphere is represented by a different color to distinguish which device the data is centered on, and corresponding to the color markers of the devices, these colored transparent spheres overlap each other in three-dimensional space.

2.2. Establishment of Single Debris Localization

The first step is to convert the given device coordinates (latitude and longitude) into a more suitable coordinate system that facilitates calculation and analysis, such as a Cartesian coordinate system. Similar approximations such as the following can be used:

Convert the longitude coordinates to X coordinates = longitude ×97304X= longitude ×97304 meters.

Convert the latitude coordinates to Y coordinates = latitude ×111263Y= latitude ×111263 meters.

Elevation (Z coordinates) directly uses the given data.

The transformation results of specific coordinates are shown in Table 2

Table 2 The result of the transformation of coordinates

Equipment	Longitude	Latitude	Elevation	Sonic Boom Arrival Time
A	10,726,890.26	3,026,798.65	824	100.767
B	10,779,337.12	3,054,836.93	727	112.22
C	10,772,720.45	3,091,442.46	742	188.02
D	10,727,863.30	3,095,892.98	850	258.985

Given the three-dimensional coordinates and time t of the sonic boom occurring in the wreck, the following equations can be established using polytropic measurement techniques for each device i:

$$\mathrm{C}(t_i - t) = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} \quad (1)$$

x_i, y_i, and z_i in this formula are the three-dimensional coordinates of the i device and the sonic boom arrival time, respectively. To solve this equation, you need to solve (x,y,z,t) in this equation.

For the solution of equations with four variables, at least four equations are needed, that is, at least four observation devices need to be arranged to complete the subsequent verification. Seven observation devices are given in the original question, so an optimization model needs to be constructed to solve the problem. Taking the sum of squares of the difference between the predicted time and the actual time as the objective function, an objective_function is defined, which accepts four variables (x,y,z position coordinates and the time t of sound burst). The predicted time of sonic boom to each device is calculated and compared with the actual time. Using the BFGS method for minimization, we find the variable values that minimize the objective_function, which represent the best estimated location and time for the source of the sound burst.

The objective function is:

$$\mathrm{f}(v) = \sum_{i=1}^{n} [t + \frac{\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2}}{c} - t_i]^2 \quad (2)$$

Where v is the vector containing x,y,z,t,t is the time of the sound burst, (x,y,z) is the position of the sound burst, C represents the speed of sound, x_i, y_i, and z_i is the three-dimensional coordinates of the i device and the time of the sonic boom arrival, and n represents the number of devices. The goal is to minimize f(v), which is the sum of squared variances of estimated and actual time.

2.3. Single Debris Localization Based on the FGS Algorithm

The BFGS algorithm (Broyden-Fletcher-Goldfarb-Shanno algorithm) is a quasi-Newton method used to solve unconstrained optimization problems. It accelerates the optimization process by progressively approximating the second-order derivatives of the objective function (the Hessian matrix). Unlike the Newton method, BFGS does not directly compute the Hessian matrix but instead updates an approximate inverse Hessian matrix iteratively, making it computationally more efficient, especially in high-dimensional spaces.

The core idea of the BFGS algorithm is to iteratively update the approximate Hessian inverse based on the gradient information from the current point and the changes in displacement and gradient from the previous iteration. The process begins by initializing a positive definite approximation of the inverse Hessian matrix (usually set as the identity matrix). In each iteration, a new search direction is calculated based on the gradient of the objective function. A line search is then performed along this direction to determine the optimal step size, and the current point is updated accordingly. Afterward, the inverse Hessian matrix is updated using the displacement vector and the gradient difference vector, generating a new Hessian approximation for the next iteration.

This iterative process continually adjusts the approximate Hessian matrix, gradually converging to a local optimum of the objective function. The BFGS algorithm exhibits good convergence properties, particularly when the quadratic approximation is effective, leading to rapid convergence. For the location of rocket debris, BFGS can predict the specific three-dimensional coordinates and time of the sonic burst by finding the location and time where the difference between the arrival time of the sonic boom and the estimated time of the model is the smallest recorded by all the observing equipment. The first eruption was calculated to have a longitude of 110.630981°, a latitude of 27.151873°, an altitude of 689.650152 m and a time of -6.530443 seconds.

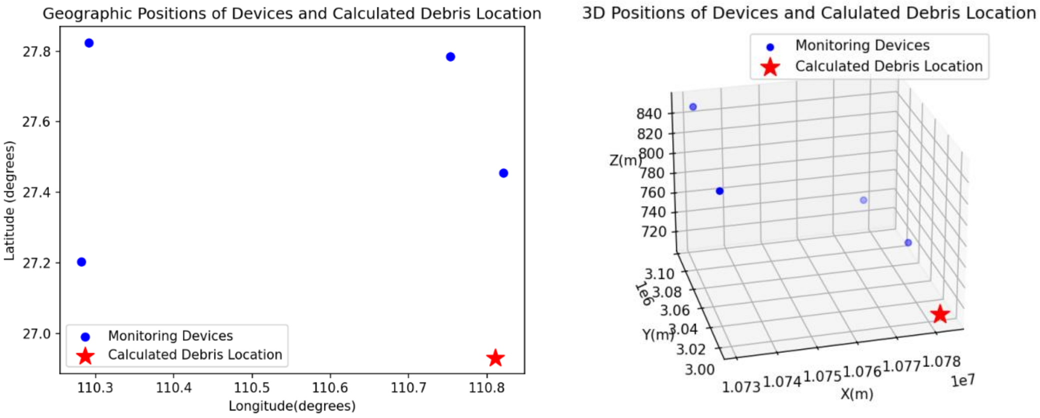

Figure 2 Visualization of the approximate location of the sound burst

In order to display the visual results more intuitively, the visual results are drawn as shown in Figure 2.

3. Synchronous Localization of Multi-source Sonic Boom Signals

3.1. Data Analysis

When the sound wave is generated, the propagation from the source of the sound burst to the observation device is a spherical wave, the radius of which increases with time, and for each debris, the sonic boom in three-dimensional space produces a uniformly variable ball of sound waves. It is the same as the data given in question 1. In order to display the given data more intuitively, the two-dimensional spherical model of the acoustic sphere and the three-dimensional spatial spherical model of the acoustic sphere are first drawn by taking A as an example, as shown in Figure 3.

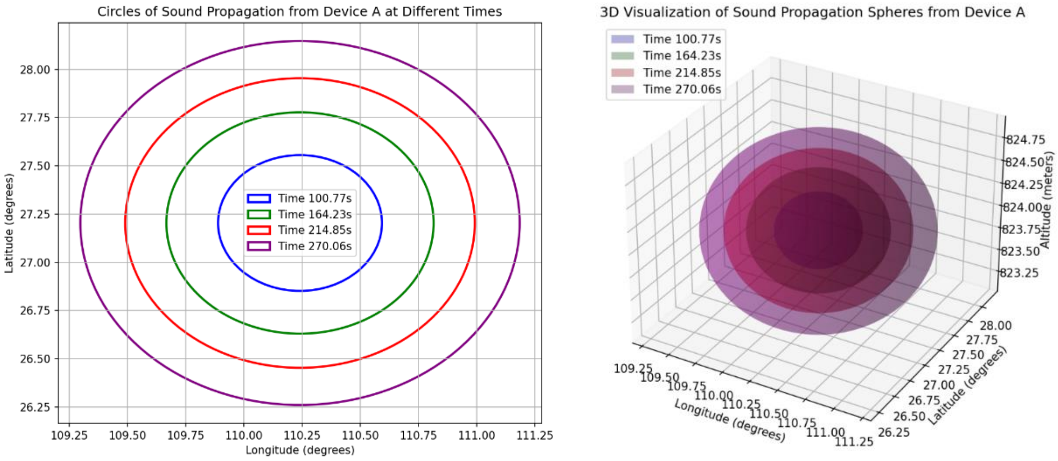

Figure 3 Device A position visualization

3.2. Establishment of a Multi-Debris Localization Model

When the sound wave is generated, the propagation from the source of the sound burst to the observation equipment is a spherical wave, and its radius increases with the increase of time. For different debris, when the sonic boom occurs in the three-dimensional space, a uniformly expanded acoustic ball will be generated. The principle of triangulation technology can be used. According to the arrival time of sound waves received by multiple observation devices, the specific three-coordinate location of each sonic boom source is determined. By comparing the time of the sound waves received by many different observing devices, the TDOA method can be used to pinpoint the specific location of each debris.

In order to determine the location of a point in three-dimensional space, at least four observation devices are required to measure from at least four different locations, but because of the presence of four more debris, at least five observation devices are required to ensure that the set sum of the only solutions that exist can be obtained (this is because, for every additional sonic boom source, the sum of the only existing solutions can be obtained). At least one additional observation device is needed to provide independent measurement data). The nonlinear minimization algorithm is used to solve the multi-variable and multi-source optimization problem, in order to find the most likely location and time of the four debris.

TDOA (Time Difference of Arrival) is a technique used to determine the position of a signal source. TDOA measures the position of a signal source based on the time difference between the propagation of the signal. In the case of multiple receiving stations, the position of the signal source relative to the receiving station can be determined by measuring the time difference between the signal's arrival at each receiving station. This principle can also be used to determine the location of rocket debris during a sonic boom, and here is the basic principle of TDOA

1) Multiple receiving stations: multiple receiving stations (at least three) are set up to receive signals from signal sources from different locations.

2) Time synchronization: Ensure that the time of all receiving stations is synchronized, so that the time difference between the signal and the station can be accurately measured.

3) Measuring the time difference: When the signal source sends a signal, each receiving station will record the time when the signal arrives. By comparing these times, it is possible to calculate the difference in time for the signal to reach each receiving station.

4) Calculate the position: Using these time differences, you can use geometric methods (such as triangle positioning) or mathematical algorithms (such as least square method) to calculate the position of the signal source.

Now assume that there are multiple observation devices (at least three) around a sonic boom source, when the sound burst occurs, the sound wave will spread out from the center point at a uniform speed. Since the distance between each observing device and the source of the sonic boom is different, the time that each observing device receives the sound wave will also be different.

For this purpose, each observation device can provide a time distance measurement proportional to the distance of the sonic boom source, and by such measurement at least three times, a system of equations can be established to solve the three-dimensional coordinate position of the unknown sound burst. Since the actual time of the signal received by each observing device is now known, it is possible to compare the time of the different signals received by the multiple observing devices, perform the difference operation, and then establish equations, which are nonlinear equations about the location of the sonic boom source, because they involve the square and square root operation for the unknown.

Our goal is to solve this equation for the location of the unique sonic boom source. In general, this can be solved using the numerical variance method, because the final analytic value is very likely not to exist or is difficult to obtain, by minimizing the error between the sum of squares between each equation, you can use the least squares algorithm to solve such problems.

In the case of multi-source sonic boom problems, such as multiple additional debris of a rocket, each debris may produce a similar equation set, then it is necessary to solve all the equations simultaneously, and also take into account the relationship between the equations, so that the specific three-dimensional coordinates and the time of sound explosion of each debris can be accurately located. Dealing with such problems requires one or more sophisticated numerical algorithm techniques.

The specific model we built is as follows:

The three-dimensional geographic coordinates (lon_i, lat_i, alt_i) of each observing device and the arrival time t_{ij} of each sonic boom recorded by them (where i is the device index and j is the debris index) are given in order to determine the three-dimensional spatial position (x,y,z) of each debris and the time t_{ij} of the sonic burst.

The sound velocity is a constant variable c. After converting the geographical coordinates of each observing device into Cartesian coordinates, the objective function can be expressed as the sum of squares of the difference between all the model estimated time and the actual time.

$$\mathrm{f}(v) = \sum_{i=1}^{n} \sum_{j=1}^{n} [T + \frac{\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2}}{c} - t_{ij}]^2 \qquad (3)$$

To take into account the constraint that the time difference between the sonic boom of all debris is no more than 5 seconds, in the objective function, you can introduce a large penalty value by adding a time difference of more than 5 seconds. If $|T_i - T_j|$>5s, a penalty term is added to the objective function.

If the expected trajectory of the wreck is known, an altitude range [min, max] can be set, and for each wreck, the elevation should satisfy $z_{min} \leq z_j \leq z_{max}$. This ensures that the outcome plan is in line with the expected flight path.

Sound velocity c will have different results under different environmental conditions, especially the change of height. A sound velocity change model c(z) about height can be introduced. The relevant formula for calculating the sonic boom arrival time should become:

$$t_{ij} = T_j + \int_{path} \frac{ds}{c(z)} \qquad (4)$$

Here, the integral is calculated along all paths from debris j to observing device i, where ds is the path element and c (z) is the speed of sound present at a certain point along that path.

Considering the effect of wind speed v_{wind} and wind direction on acoustic wave propagation, the actual acoustic wave propagation time t_{ij} from debris to observation equipment should also consider the effect of wind.

$$t_{ij} = T_j + \int_{path} \frac{ds}{c(z)+v_{wind} * u_{ij}} \qquad (5)$$

Where u_{ij} is the unit vector from debris j to observation device i, and the dot product is expressed as a vector dot product to calculate the X-axis projection of wind speed in the direction of sound wave propagation.

The final optimization model is as follows.

$$minf(v) = \sum_{i=1}^{n}\sum_{j=1}^{n}[T + \frac{\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2}}{c} - t_{ij}]^2 \quad (6)$$

3.3. Multi-Debris Localization Based on Genetic Algorithms

Genetic Algorithm (GA) is an optimization technique inspired by natural selection and genetics. It starts with a population of individuals, each representing a potential solution to a problem. These individuals are evaluated based on a fitness function that measures solution quality. Higher fitness individuals are more likely to be selected for reproduction. Through crossover, genetic information from two parents is combined to produce offspring, and mutation introduces small random changes to maintain diversity. This process of selection, crossover, and mutation is repeated over multiple generations, gradually evolving the population towards better solutions. The algorithm continues until a stopping criterion, such as a maximum number of iterations or an acceptable fitness level, is met. GA is effective for solving complex, multi-dimensional problems due to its ability to search globally and avoid local optima, though it can be sensitive to parameter settings like population size, crossover, and mutation rates.

The final results of multiple debris location are shown in Table 3.

Table 3 Multiple debris location results

Debris	Longitude	Latitude	Altitude	Time
1	110.464651	27.681036	8.9	95.77
2	110.474550	27.647701	821.7	95.77
3	110.450770	27.664188	1528.0	95.77
4	110.491338	27.604971	6075.6	100.77

In order to display the visualization effect more intuitively, the three-dimensional visualization diagram is drawn as shown in Figure 4.

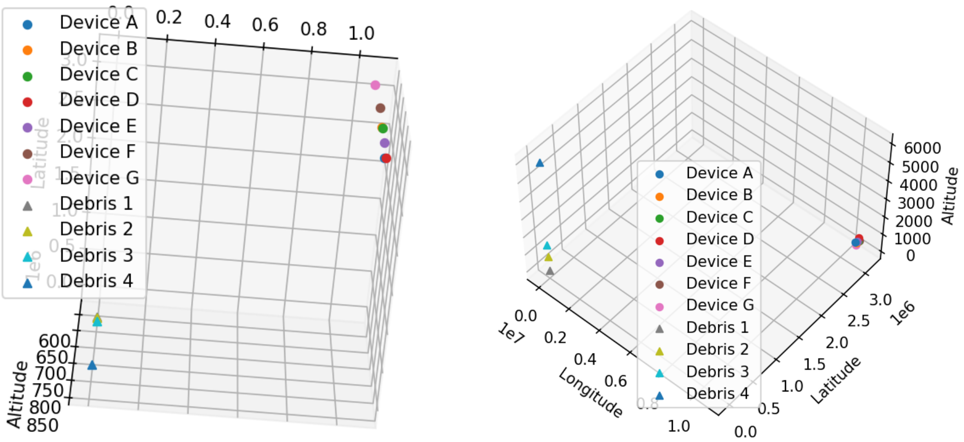

Figure 4 Visualization of multiple debris location results

After processing by the optimization algorithm, the locations of the four pieces of debris are respectively located in different three-dimensional space points, forming a discrete distribution, and the location of each piece of debris is determined by its corresponding latitude and longitude and elevation. As can be seen from the figure, wreckage No. 4 is obviously in a higher altitude value than other debris, which may represent a different flight trajectory or have disintegrated. The time of all the debris is infinitely close to 95.77 seconds, except for the fourth debris, its time is 100.77 seconds, just compared with other debris, in line with the previous 5-second error expectation, as long as the sonic boom time difference does not exceed 5 seconds, it conforms to the constraint term.

Through these diagrams, the operation of the detection grid for the recovery of rocket debris or the residual value of research, development and utilization can be summarized, and the three-dimensional spatial coordinate relationship between the optimized predicted location of debris and the actual observation equipment can be provided. The successful optimization results provide accurate coordinate data support for the on-site and subsequent debris recovery teams.

4. Conclusion

This study has successfully developed and validated a sonic boom localization model for rocket debris based on the BFGS algorithm and genetic algorithms. By converting the geographical coordinates of observation equipment into a Cartesian coordinate system and utilizing multi-lateration techniques, the model effectively predicts the three-dimensional coordinates and occurrence time of the sonic boom. The BFGS algorithm demonstrated good convergence during the solution process, while the genetic algorithm showed superior global search capabilities in handling the synchronous localization of multi-source sonic boom signals. Three-dimensional visualization results further confirmed the accuracy and reliability of the model's predictions. Although positive results have been achieved, there is still room for improvement in the model. Future research can explore more factors affecting the propagation of sound waves, such as the inhomogeneity of the atmospheric layer structure and wind speed variations, to further enhance localization accuracy. Additionally, parameter tuning and optimization of the algorithms are important directions for improving model performance. With continuous optimization and adjustment, the model is expected to play a greater role in the fields of rocket debris recovery and related safety assessments.

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