

Model-based dragon ecological analysis

Jiamian Wu

North China Electric Power University, Hebei, Baoding, 071000

Email:hibiscus007@126.com

Keywords: Ecological requirements, Dragon's energy model, AHP, Logistic Function, RGA Model

Abstract: If the dragons in the novel live in our world today, how can we human get along well with the dragons under the premiss of keeping the ecological environment stable? A lot of problems are waiting us to address. In order to analyze dragon characters, behavior, habits and interaction with environment, we build models to discuss their necessary caloric intake, energy expenditures and the most important factors during its healthy growth, these factors' impact and influencing results as well. In the end, we measure its growing area through the analysis on the above factors.

To calculate the caloric intake and energy expenditures for a dragon, we propose a reasonable assumption that dragon's weight grows in line with their age. According to the principle of conservation of energy, we structured a differential equation of first order to describe the process of energy transformation, including energy intake, expenditures, weight, activity intensity, whether got injured and extent of its wound and then establish Model 1: Dragon's energy model. This model shows that the necessary energy is a little when the dragon is young, but with the increase of its weight, the caloric intake and energy expenditures will rise sharply. In addition, at the same change of activity intensity, the older the dragon is, the greater variable quantity of energy intake is.

Next, we consider all kinds of factor affecting the dragon's growing area synthetically. Then we find the weight of weather condition reaches 25% through AHP, taking advantage of which, we screen three the most important elements influencing its growing area: precipitation, food per unit area, and river discharge per unit area. The calculating result of MATLAB presents, their weight arrives at 83.7%. Then we build Model 2: Resource-dependent Growing Area Model (RGA) considering these three factors to measure the supporting area for three dragons. Next, we use Logistic Function to build the relationship between the food per unit area and fresh water resource (precipitation and river discharge per unit area) simplifying the calculation. We find the growing area for three dragons is not stable, and it depends on the dragons' body type (the heavier dragons are, the more energy and fresh water resource they need, which leads to larger growing area) and local resources (the less biological and fresh water resources this region has, the larger growing area they need.)

Based on the conclusion from AHP: the weather condition has great impact on the dragon's growth. We analyze the supporting areas on arid regions, warm temperate regions and arctic regions. We think the rank of areas from large to small is warm temperate regions, arctic regions, arid regions]. Some arid regions and arctic regions even aren't suitable for the growth of dragon. The dragon needs to live in the places with abundant specious and amply fresh water resource. Meanwhile, it is sensitive to the temperature, vegetation coverage and degree of air thinness that it can't stand living in high altitude areas and the polar region for a long time.

Finally, we analyze the performance of our model and the sensitivity of our model, proving that our model is relatively stable for different parameters.

1. Introduction

1.1 Problem analysis

The ecology problem has aroused wide concern. Only when human and animals live in harmony, our world can develop steadily and rapidly.

In A Song of Ice and Fire, a great fantasy novel, the mother of dragon Daenerys Targaryen and her three dragons, Drogon, Rhaegal, Viserion are the most critical roles undoubtedly. The dragon exists reasonably in the fabled world. However, if the dragons live in our real world today, what is their situation? A lot of problems are waiting us to address.

Task 1: Measure the energy expenditure of a dragon and its caloric intake requirement.

Task 2: Explore how important are the climate conditions to our analysis on the dragon’s growth.

Task 3: Estimate how much area is required to support the three dragons.

1.2 Our work

The dragon is so large that it probably has high demand for growth. If it lives in our life with humans, we should think over a plenty of factors, like energy it needs, climatic conditions, terrain prerequisite and so on.

Firstly, based on the essential research on animal’s energy intake and expenditures, we established dragon’s energy model, considering the relationship among its weight, energy intake and expenditures.

Next, thinking over that we can’t synthesis every finely-divided impact elements, we built a screening model to determine three most vital factors which influence the dragon’s growth.

Eventually, we used Logistical Function to assure the relation among these influencing elements, and then took advantage of fresh water resources per unit area and the water intake for a dragon to measure these three dragons growing area.

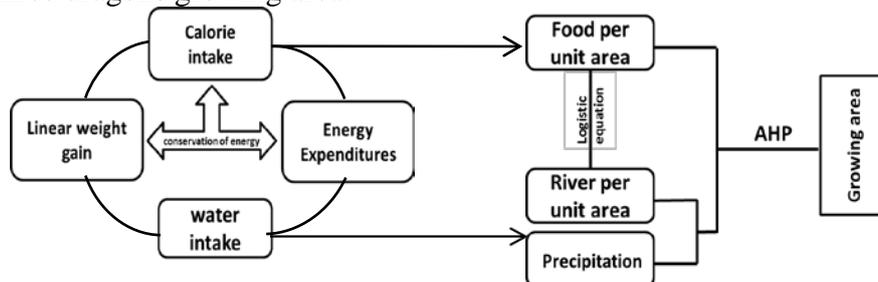


Figure 1. Process flow for model creation

2. Assumptions and Symbols

2.1 Assumptions of the initial data

1) We assume that the dragon’s weight (W) increases on line, satisfying: $W=0.07t+10$, with t time (day).

Reason: When hatched, the dragons are small, roughly 10kg, and after a year grow to roughly 30-40kg. They continue to grow throughout their life depending on the conditions and amount of food available to them. Thus, we assume that the dragon’s weight (W) increases on line and neglect all the leap years (365 days a year) to predigest the problems.

2) Some assumptions about dragons: the dragons are able to fly great distances and can fly to any place it wants, breath fire, resist tremendous trauma and live for 200 years.

Reason: Respect the original work, A Song of Ice and Fire.

3) We think the dragon feeds on meat on the top of food chain.

Reason: As a so large animal, the dragon can defeat any other animals. Meanwhile, only meat can provide the dragon with enough energy.

4) We suppose the dragon is a kind of homothermal animal with the most appropriate temperature is 24°C and has similar living environment to human.

Reason: In the book, the dragons live with human together. And the pterosaur is also homothermal in reality.

2.2 Symbols and definitions

Symbols	Definitions
W	weight of the dragon
E _{intake}	energy intake
E _{expenditure}	energy expended in various ways
t	time (day)
T	temperature of living environment
α	activity intensity
β	coefficient of trauma
$\varepsilon(x)$	judgement of injury
C _i	affecting factors, i=1,2,3,4,5,6
λ	maximum eigenvalue
B _j	judgement matrix, j=1,2,3,4,5,6
N	number of creatures per unit area
K	environmental capacity
P	initial data about the creatures' number per unit area
W _f	fresh water resources per unit area
W _t	water intake per day
S	area required to support the three dragons

3. Models and Results

3.1 Model 1: Dragon's energy model

3.1.1 The energy intake

Being assigned to analyze dragon characters, behavior, habits and so on, we set about our work from its diet, its energy intake and expenditures. According to the conservation of energy, Energy balance, through biosynthetic reactions, can be measured with the following equation:

$$\text{Energy intake} = \text{Energy expended} + \text{Change in stored energy}^{[1]}$$

We use a linear first-order differential equation to establish an energy model, assuming that there exists a function W(t), which can make that any positive integer all meet W(n)=the dragon's weight at the nth day. From a long view, it is a continuously differentiable function.

$$\frac{dW}{dt} \approx \frac{\Delta W}{\Delta t} = \Delta W \Big|_{\Delta t=1}$$

Moreover, we suppose that 1000k calorie will result in gaining weight 1 kg.^[2]

$$E_{\text{intake}} = E_{\text{expenditure}} + 10^4 \frac{dW}{dt} \quad (1)$$

Where: E_{intake} is the energy intake; $E_{\text{expenditure}}$ is energy expended in various ways; W is dragon's weight

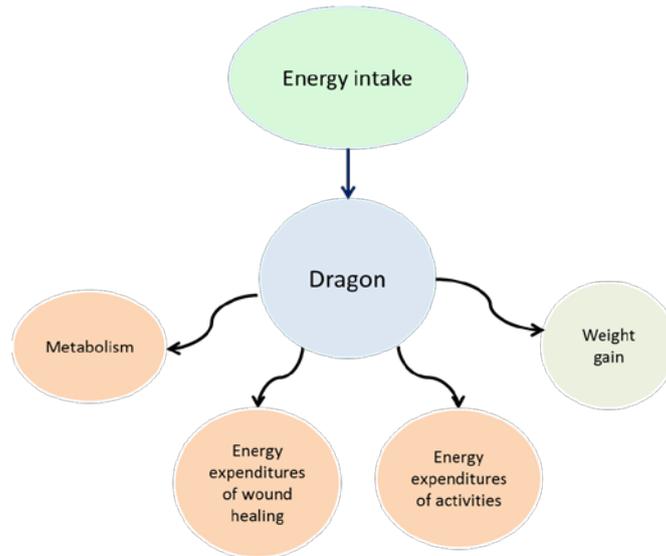


Figure 2. The energy flow

3.2 The expenditures of energy

Besides, the expenditures of energy is relative to the biological metabolism and activity intensity. On the background of dragon, we also take its particular function into consideration, resistance of trauma, at the same time. Based on the assumption that the dragon is a kind of homothermal animal, the dragon will need more energy if it lives in a cold environment.^[3] So the relationship between temperature and energy expenditures is negative.

$$E_{\text{expenditure}} = [24W + \alpha W + \beta \varepsilon(x) W] / T \quad (2)$$

Where: α is the activity intensity; β is a coefficient of trauma; T is the temperature of environment =24°C

$$\varepsilon(x) = \begin{cases} 1, & \text{if the dragon was injured} \\ 0, & \text{if the dragon was not injured} \end{cases}$$

3.3 Model 1 Results & Analysis

Combine (1) and (2) equation to get an integrated equation:

$$E_{\text{intake}} = [24W + \alpha W + \beta \varepsilon(x) W] / T + 10^4 \frac{dW}{dt} \quad (3)$$

Solve the equation (3) and associate its result with our hypothesis of the dragon's weight, finally get the relationship between the energy intake and time.

$$\begin{cases} E_{\text{intake}} = [(0.07t+10)-10\exp(-M/10^4T)] M / T \\ M = 24 + \alpha + \beta \varepsilon(x) \end{cases} \quad (\beta = 0)$$

In Figure 3, we omit the energy expenditure of wound healing and set α from 0 to 10, then discuss the minimum amount of energy required for growth at a given age.

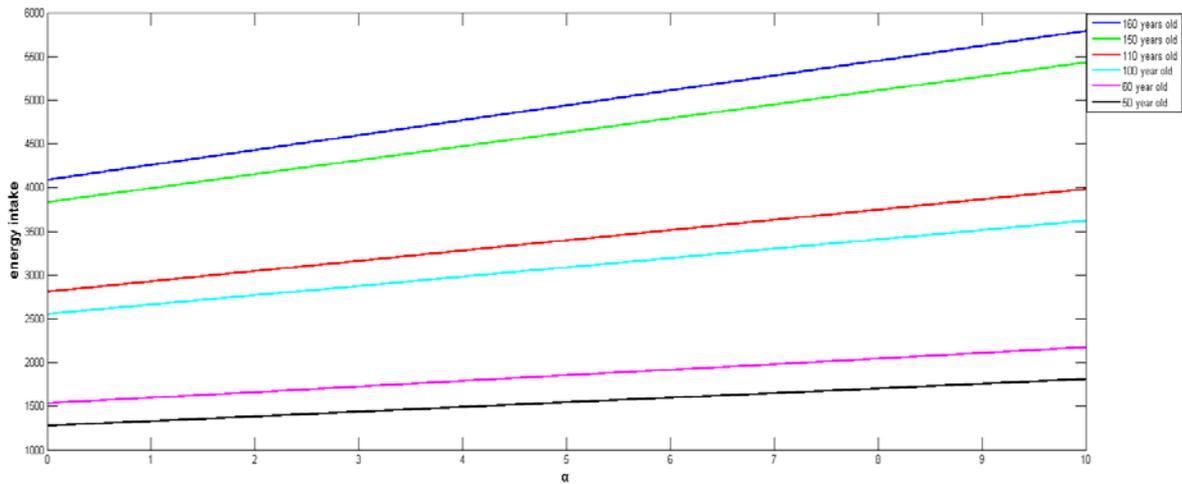


Figure 3. Energy Intake model imageI

The graph shows older the dragon is, the more energy it intakes. And at the same change of activity intensity, the older the dragon is, the greater variable quantity of energy intake is.

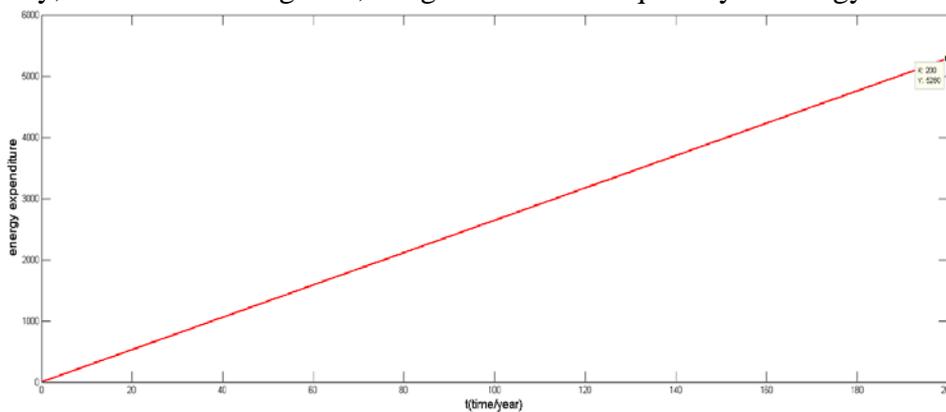


Figure 4. Energy intake model imageII ($\alpha = 0$)

Compared with Figure 3, we can find that the energy expended in healing the wound has little relation with age. What's more, the amount of calories a dragon consumes to satisfy the need of growing increases with the rise of its weight. There are significant differences in the amount of calories a dragon needs at different ages and activity intensity levels. This model is not only applicable to dragons, but also other animals in the real world, which is of great significance for the breeding of animals in zoos and the protection of endangered animals.

3.4 Model 2: Resource-dependent Growth Area Model (RGA)

3.4.1 Screening Model Based on Analytic Hierarchy Process (AHP)

The Analytic Hierarchy Process (AHP) is a structured technique for organizing and analyzing complex decisions, based on mathematics and psychology. It has particular application in group decision making, and is used around the world in a wide variety of decision situations.^[4]

When discussing the supporting area for dragons, we believe it is mainly affected by six major factors, temperature, precipitation, river charge per unit area, food per unit area, degree of air thinness, vegetation coverage. In view of that AHP can analyze hierarchically, we use it to determine the more critical elements from them.

Firstly, establish a hierarchy model.

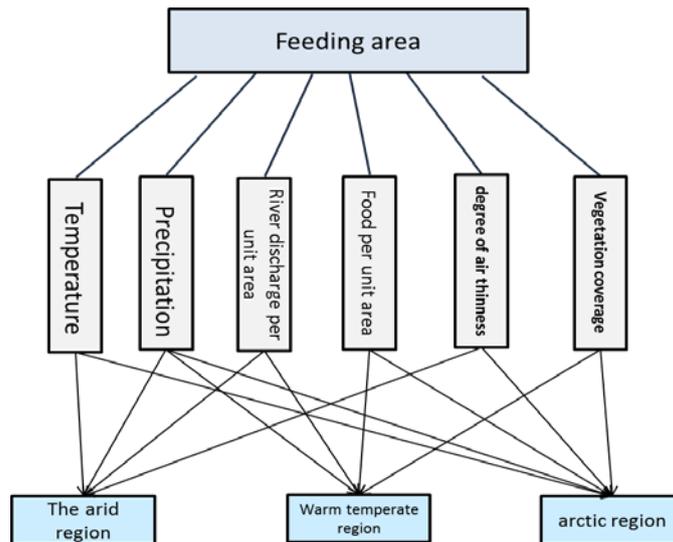


Figure 5.Hierarchy model

Secondly, construct a paired contrast matrix and single hierarchical order, and then get its maximum eigenvalue with corresponding eigenvectors.

- C_1 is the Temperature
- C_2 is the Precipitation
- C_3 is the River charge per unit area
- C_4 is the Food per unit area
- C_5 is the Degree of air thinness
- C_6 is the Vegetation coverage

Where:

$$A = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{matrix} & \begin{pmatrix} 1 & 1/6 & 1/7 & 1/7 & 2 & 1 \\ 6 & 1 & 3/4 & 1/3 & 5 & 2 \\ 7 & 4/3 & 1 & 1/2 & 6 & 3 \\ 7 & 3 & 2 & 1 & 8 & 5 \\ 1/2 & 1/5 & 1/6 & 1/8 & 1 & 1 \\ 1 & 1/2 & 1/3 & 1/5 & 1 & 1 \end{pmatrix} \end{matrix}$$



$$\lambda_{\max} = 6.2242$$

Table 1. AHP production

Factors	Weight
Temperature	0.0530
Precipitation	0.1885
Food Per Unit Area	0.4031
Vegetation Coverage	0.0674
Degree Of Air Thinness	0.0427
River Discharge Per Unit Area	0.2454

From Table 1, it can be seen that the sum weights of Precipitation, River Charge Per Unit Area

and Food Per Unit Area have reached 83.7%, which are the most vital factors affecting the growing area of dragons. At the same time, different climatic conditions will cause different precipitation and temperature, with weight of 25%. So we can find that climatic conditions play an important role.

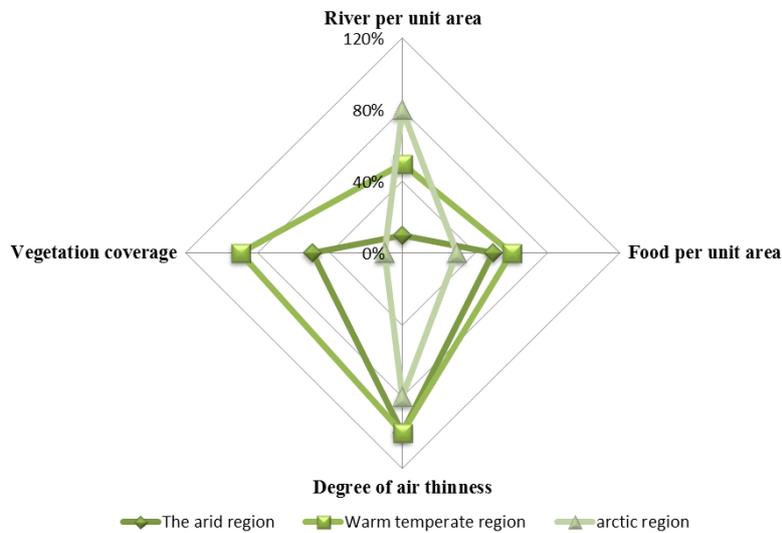


Figure 6. Area chart about affecting factors

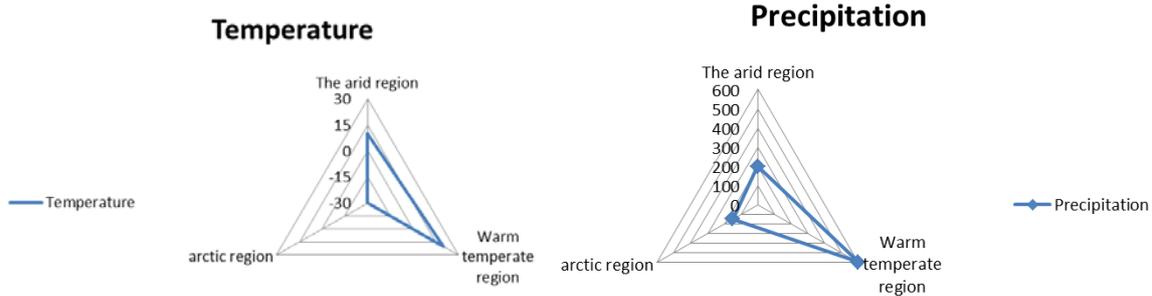


Figure 7. Area chart about temperature

Figure 8. Area chart about precipitation

From these three radar maps, we can get six criteria judgement matrix about three regions, the arid region, warm temperate region and arctic region.

	Temperature	Precipitation
	Temperature	River discharge per unit area
	Precipitation	Vegetation coverage
	River discharge per unit area	
	Vegetation coverage	
	Food per unit area	

$$B_1 = \begin{pmatrix} 1 & 1/3 & 2 \\ 3 & 1 & 5 \\ 1/2 & 1/5 & 1 \end{pmatrix}$$

$$B_2 = \begin{pmatrix} 1 & 1/4 & 1/2 \\ 4 & 1 & 2 \\ 2 & 1/2 & 1 \end{pmatrix}$$

$$B_3 = \begin{pmatrix} 1 & 1/2 & 1/3 \\ 2 & 1 & 1/2 \\ 3 & 2 & 1 \end{pmatrix}$$

$$B_4 = \begin{pmatrix} 1 & 1/2 & 2 \\ 2 & 1 & 4 \\ 1/2 & 1/4 & 1 \end{pmatrix}$$

$$B_5 = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 1/2 & 1/2 & 1 \end{pmatrix}$$

$$B_6 = \begin{pmatrix} 1 & 1/4 & 4 \\ 4 & 1 & 7 \\ 1/4 & 1/7 & 1 \end{pmatrix}$$

In order to simplify the calculation, we use MATLAB to calculate the eigenvector. The eigenvector of each attribute can be obtained as:

$$W_1 = \begin{pmatrix} 0.2297 & 0.1428 & 0.1624 & 0.2857 & 0.4000 & 0.2290 \\ 0.6483 & 0.5715 & 0.2969 & 0.5715 & 0.4000 & 0.6955 \\ 0.1220 & 0.2857 & 0.5396 & 0.1428 & 0.2000 & 0.0755 \end{pmatrix}$$

3.4.2 Screening model's result and analysis

The combined weight of the three regions on the target is $W1 * \text{Weight}$ (Table 1).

$$W_2 = \begin{pmatrix} 0.227 \\ 0.509 \\ 0.264 \end{pmatrix}$$

In the section above, we use AHP to screen three factors, which are more important comparing with others. They reflect what is needed for a dragon's growth. Since they need huge amount of energy and breathe flames, so a lot of oxygen, enough prey, appropriate environment with abundant species are necessary. Droughty, high altitude and desert areas are not suitable for the growth of dragons. Synthesize Model 1, the normal growth of dragons requires huge energy, so it has strict requirement on food, fresh water and oxygen resource. At the same time, it also causes serious consumption of resources and may cause the degradation of ecological environment. [5]

Comparing the weight vectors of arid regions, warm temperate regions and arctic regions, we find that warm temperate regions are more suitable for the growth of dragons. When the dragon moves to a drought region, the lack of precipitation will affect it. When the dragon moves to the arctic regions, it will lack food due to the deficiency of local species, which will also affect the dragon's healthy growth and its ability to resist trauma and breathe fire

3.4.3 Area Model based on Logistic Function

Based on the Analytic Hierarchy Process about the growth of dragon, we can find that precipitation, river discharge per unit area and food per unit area are the most vital elements affecting the dragon growth area. The sum of their weights has reached 83.7% (Table 1), so we take these three factors into consideration when we are assigned to measure its growing area.

In order to simplify the problem, we regard precipitation and river discharge per unit area as the component of fresh water resource in a region. According to the biology, when a population live in a limited space, the competition for the resource will definitely being more fierce with the rise of its density, then it will influence its reproduction rate and survival rate, even reduce its natural growth rate, and eventually this population stops growing.

The amount of creatures in a region and fresh water resource also have such relationship, that is, the growth rate of creature will increase with the augment of fresh water. For the dragon, Food per unit area will also increase with it. However, when the number of species reaches a certain sum, the competition for survival and deficiency of other resources will affect their reproduction rate and survival rate, so their growth rate will be lower. It means that Food per unit area have "s" relationship with the fresh water resource.

Logistic Function is a common s-shape function, which is an optimal model to describe the law of population growth under the condition of limited resources. Therefore, Logistic Function can describe this relationship very well.

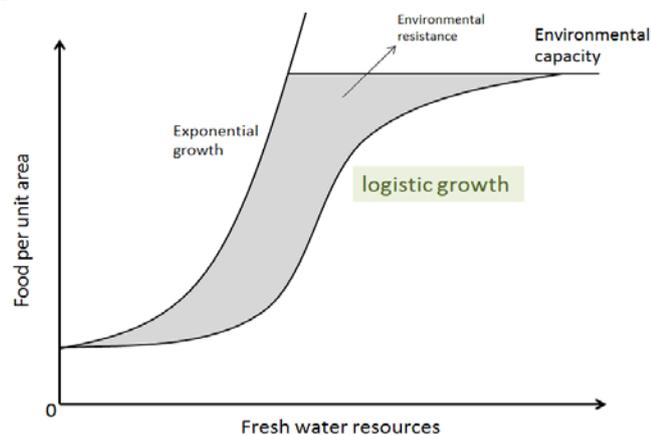


Figure 9. The relationship between food per unit area and fresh water resources [6]

The expression based on the Logistic Function [7] is:

$$N = \frac{KP e^{W_f}}{K + P(e^{W_f} - 1)}$$

Where: N is the number of creatures per unit area; K is the environmental capacity; W_f is the fresh water resources per unit area; P is the initial data about the creatures' number per unit area.

To support three dragons, we suppose that they are relatively independent, neglecting the problem of their propagation. In nature world, the efficiency of energy transfer is about 10%-20%. [8] We set this efficiency at 20% to simplify the problem. So a dragon needs to intake $E_{intake}/0.2 = 5E_{intake}$ at least. As for three dragons.

In addition, water is the source of life for creatures, the water intake for a dragon is related to its weight and activity. We assume that the water intake per day for a dragon is W_t, so the total water

intake for these three dragons is $\sum_n W_t^n$ and the fresh water per unit area is W_f. Thus, so as to meet

$$S = \sum_n W_t^n / W_f$$

the basic growth of three dragons, the minimum supporting area

3.4.4 Area Model's result & analysis

Above on, the area required to support the three dragons should satisfy:

$$\left\{ \begin{array}{l} S = \max \left[\frac{\sum_n 5E_{intake}^n}{N}, \frac{\sum_n W_t^n}{W_f} \right] \\ N = \frac{KP e^{W_f}}{K + P(e^{W_f} - 1)} \end{array} \right.$$

Where: S is the area required to support the three dragons; E_{intake}^n is the nth dragon's energy intake, W_t^n is the water intake per day for nth dragon

This Area Model shows that the older dragons are, the more energy they need, the larger supporting area is required. Meanwhile, the supporting area is also related to the local region's condition. If this region is rich in fresh water, the required area will be small relatively.

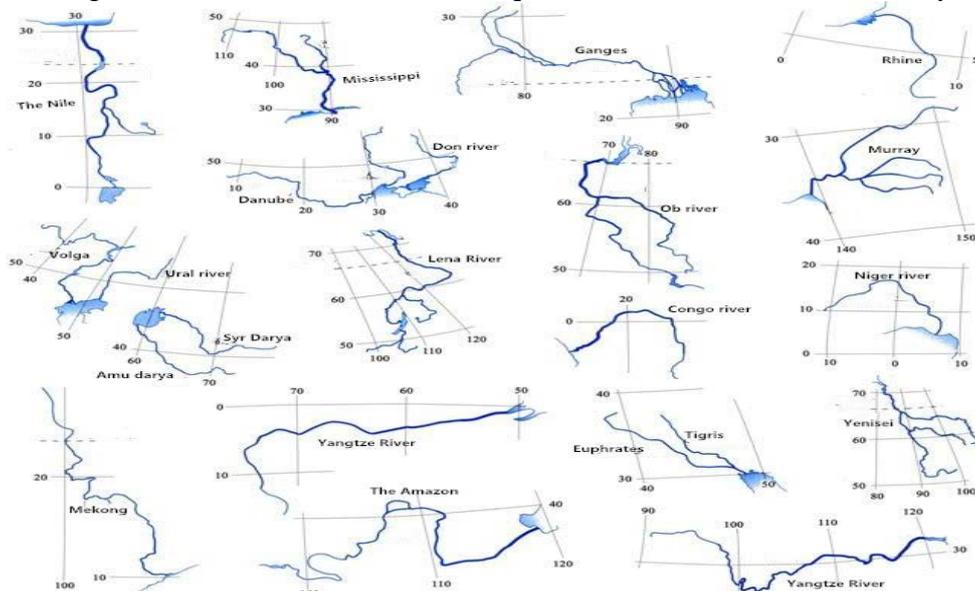


Figure 10 .major rivers of the world [9]



Figure 11. Major river basin of the world [10]

4. Sensitivity Analysis

We think the result we got must be insensitive and the energy intake remain the same approximately, and the energy intake depends on its weight to a large extent. $\beta = 0.81$ or 0.80 , and observe its energy intake judging its sensitivity.

4.1 Sensitivity analysis of Model 1

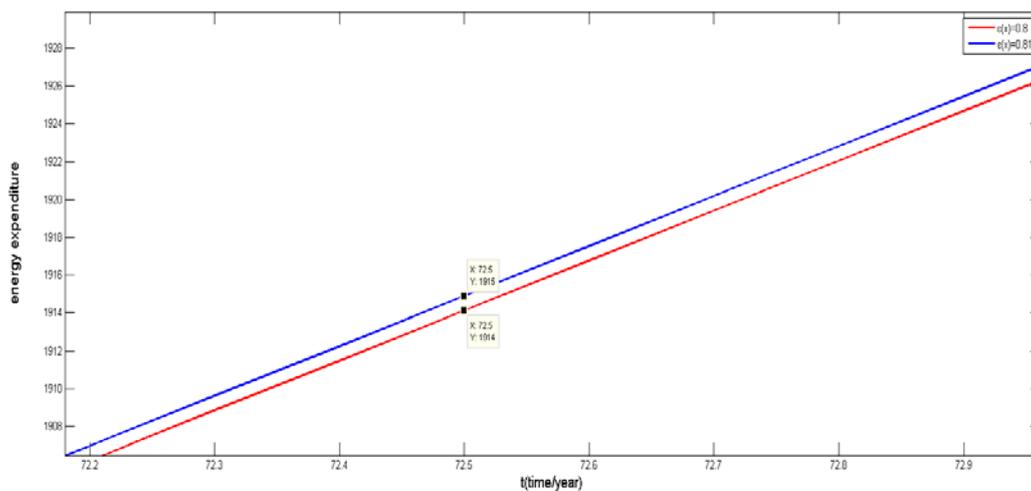


Figure 12. Sensitivity analysis on β

Table 2. Table Sensitivity analysis

β (weight=15kg)	0.80	0.81
Energy intake	1914	1915
sensitivity	0.041797283	

The Sensitivity Index shown above means that when the β increased by 1%, the energy intake will in turn increase by 0.042%. So, the variation of this index has minor influence on the results.

5. Strengths and weaknesses

5.1 Strengths

Under the basis of vast analysis and careful studies, we think over the special features of dragons

and many kinds of factors on its growing environment to make our model more accurate.

On the second model, we combine Analytic Hierarchy Process with Logistic Function. What's more, we take the interaction between biology in reality and different variables.

Area Model depending on the fresh water resource which was initiated by us can explain what size is needed to support the dragons very well. This model can be used in measuring the growing area not only for the dragons but other species.

5.2 Weaknesses

We assume the dragon's weight increases on line, which is not very precise to some extent.

In measuring the growing area, we only analyze the minimum area for these three dragons when they are supported in different regions in qualitative, so that we can't quantitatively determine the specific value for lack of explicit data.

On our models, we don't consider the destruction of fire jetted by the dragon.

In reality, since the dragon is imaginary, we can't do accurate calculate on its characters to establish models.

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