Application of Pde in Image Segmentation

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Abstract: Image Segmentation is a Basic and Important Subject of Image Analysis and Computer Vision. the Goal of Image Segmentation is to Divide the Image Region into Several Separate Parts. in Each Sub Region, the Specific Properties of the Image Represent Consistency. Pde Method is Widely Used in the Field of Image Segmentation. the Basic Idea is That the Level Set Function Evolves under the Control of Pde (Initial and Boundary Conditions). the Zero Order Set of Pde Solution Gives the Target Boundary. the Evolutionary Partial Differential Equation of the Order Set Function is Obtained by Minimizing the Energy Function. This Method is Called Variable Hierarchical Collective Method.

1. Introduction

In the Process of Image Processing Research, People Are Often Interested in Specific Parts of the Image. These Parts Are Often Referred to as Objects of Interest or Regions of Interest (Roi). the Important Purpose of Image Processing is to Take the Objects in the Image as Objects. the Purpose of Analysis is to Obtain Objective Information Related to the Target and Establish the Relevant Description of the Image [1]. Therefore, in an Image, It is Called Image Segmentation to Separate the Target from the Background for Subsequent Processing. More Precisely, Image Segmentation Means That the Image is Divided into Several Non Overlapping Regions Based on Functions Such as Gray Level, Color, Texture and Shape. as a Result, These Functions Show the Similarity or Consistency of the Same Region, and There Are Obvious Differences between Different Regions.

2. The Research Status of Partial Differential Equation Image Segmentation

PDE (partial differential equation) image segmentation is a nonlinear image segmentation method developed in 1980s. The internal force of the curve itself and the external force of the image data make the curve move towards the target contour, so as to obtain the characteristics of the target object. The biggest advantage of PDE image segmentation is that the curve keeps continuity and smoothness in the process of continuous evolution, which can achieve continuous and closed contour extraction [2]. As a relatively new and effective image segmentation method, PDE based image segmentation technology provides a good way to solve many problems of existing image segmentation technology.

In 1987, Kass, Witkin and terzopoulos proposed the snake shaped contour model, which uses the continuous curve model to find the image edge directly from a new viewpoint, and defines the initial evolution curve with energy near the image area (explicit). Under the combined action of its internal force and external force generated by image information, it moves continuously and finally converges to the edge of the object. In this model, variational method is introduced for the first time, and its local extremum constitutes the image segmentation result which can be processed with high level image. Like this, when looking for important image functions, we can compare them with the model by pushing the image functions to the appropriate local extreme points. However, this model has several disadvantages.

It is necessary to set the initial curve near the edge. The curve can not adaptively change the topology in the evolution process, so as to segment multiple targets at the same time. In 1988, the level set method proposed by R. Osher and jasethian further improved the active contour model.
This method has been applied in optimization control, computer graphics, material science and other fields. The use of level setting method in image processing and computer vision began in two well-known literatures of caselles et al. In 1993, Malladi and his colleagues use level sets to represent snakes, which implicitly represent curves as a level set on a higher dimensional surface (level set function) [3]. Unlike the parametric model, the level set method does not track the movement of the curve at different times, but in order to simulate the evolution of the curve, the level set function at different times is updated in the fixed coordinate system. This method has greatly expanded the scope of application of named model and greatly developed the theory of action contour model.

The segmentation method of partial difference image model unifies image, initial contour, target contour and constraints. Considering the initial curves and surfaces in the image space, it also defines the internal energy related to the curve or surface shape and the external cans related to the image. Among them, the internal energy controls the smoothness and continuity of the curve and surface, and the external energy is generally related to the edge features. Under the compound effect of internal energy and external energy, the contour is deformed, and finally the continuous edge of the image is obtained. Due to the diversity and complexity of image types, it is difficult to determine the appropriate threshold size for threshold based discrimination methods. Edge detection method is not ideal to distinguish noisy image, blurred edge and texture image. The method of region based partition is usually complex. The image processing method based on PDE has become the most interesting topic for researchers in related fields.

![Fig.1 Segmentation of an Artificial Image](image)

### 3. Research Based on PDE

Using the idea of image partition based on PDE, we generalize and define the curve of image according to the characteristics of image contour and image characteristics[4]. The curve (also called active contour) and all the characteristics of its motion can be described by the energy function guided by the physical motion system. Through this energy function, the curve energy in the process gradually reduces to the minimum, and finally stops near the contour of the object.

The method of representing active contour is level set method. In this method, the development of two-dimensional (three-dimensional) closed curves (surfaces) is solved as a linear equation of the development of series function surfaces in three-dimensional (four-dimensional) space. However, the increase of size will inevitably lead to the increase of calculation amount, and the slow calculation speed is its main disadvantage. In order to improve the calculation speed of level set, two numerical methods of high-speed evolutionary level set method are widely used in the numerical calculation of geometric action profile model: the band method and the high-speed mark method. Later, the method of calculating the plane distance of the composition function of the symbolic distance function using Setan's al. High speed symbolic method was proposed, and Tsai proposed the source scanning method. The average curvature motion model of this section mainly adopts the average curvature motion model. Inspired by this model, the model of this paper is proposed. The existing active contour models are all obtained from the average curvature motion,
and all image gradients are used to stop the evolution process.

In order to ensure that the horizontal set function $\varphi(T, x, y)$ is not flat or too fast in the motion curve $C(T)$, it is necessary to reinitialize the function $\varphi(T, x, y)$ into the symbolic distance function periodically. In other words, in order to solve the following partial differential equations relative to the average curvature motion equation, a new shadow contour model independent of the slope of the image is proposed. In this model, the zero level set function can separate the target background in the image.

The first term (curvature term) is the zero level setting line of the regularization function $\varphi$. It plays an important role in smoothing the edge control curve of the complex target area and avoiding the isolated small areas (such as noise points) in the final segment. Because the actual image noise will affect the image quality more or less, especially for the noisy image, the curvature term must be included. The second function is to quickly drive the level setting function $\varphi$ and adaptively move to the opposite direction between the target and the background. When the initial contour $0 \varphi(x, y)$ is properly selected, the zero level set line of the level set function $\varphi(T, x, y)$ can quickly reach a stable state, which means that the target background can be separated. (get the final split).

In the first experiment, the horizontal lines in the horizontal direction are tested to reach the same stable state from different initial positions. Starting from three different initial functions, the model shows that it can achieve ideal segmentation results. The expansion of the zero level set $\varphi(T, x, y)$ of the expression of the three line image representation in Fig. 6.1, wherein the first column is initialized as the mathematical symbol distance function, and the second and third columns are initialized as $\rho = 1$. The first line represents the initial curve, the second line represents the initial level setting, the third line represents the final segmentation result, and the fourth line represents the final level setting function. Each initial state gets the same final segmentation configuration. Other binary images are tested and the same segmentation results are obtained.

4. Conclusion

In the field of image processing and computer vision, many variational and partial differential equation models have been generated, and great progress has been made in image processing and computer vision. These models with higher-order prior knowledge are helpful to integrate data constraints and prior knowledge in the activity prospect. Taking the image segmentation model of PDE as the center, the following results are obtained.

A symbol level setting method for solving Mumford-Shah segmentation model is proposed. Compared with the previous level set method, the symbolic level set method has the following advantages. This method is suitable for the well-known Mumford-Shah segmentation model, and the derived model is adopted by ICT center of Chongqing University. Disadvantages: symbolic level set method is suitable for solving classes of variable level set model.

A class mixed gradient method for solving variable hierarchical set model is proposed [6]. Different from the existing partial differential equations, the gradient of all-round function is calculated [7]. The hybrid gradient method has the advantages of internal energy and external energy hybrid gradient method, which can be used to solve a class of variable classification set model with high efficiency.

In order to compare with the existing correlation model (RSF model, LIF model), a variable classification set model is proposed, which can segment the gray level uneven image effectively [8]. The biggest advantage of this model is that it can initialize the level set function to zero function, and it will not display. Selection of initial contour.

A partition model of PDE based on mean curvature motion is proposed. The main advantage of this model is to overcome some shortcomings of the existing edge model, because it is only based on the region, and does not need image gradient as the condition to stop evolution [9]. By solving the partial differential equation in numerical way, the difference scheme in the wind is adopted in the existing model. The model can also effectively use the semi negative addition operator separation (AOS) method.
References


