Kane Dynamics Modeling of Large Flexible Cantilever Plates

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Keywords: plate; Kane method; dynamics modeling; primary dynamics model

Abstract: In this paper, for the flexible plate attachment, the cantilever plate is selected as the research object, and the Kane dynamics method is used to model the "one-time dynamic model" of the flexible attachment, and the power of the flexible plate attachment under the wide range of motion conditions is obtained to learn the equation. On this basis, compared with the modeling results of “zero-order dynamics model”, the significance of modeling using Kane method is illustrated.

1. Introduction

The theoretical methods of classical rigid body dynamics mainly include Newton's Euler method and Lagrangian method, which have been developed for more than 200 years. The classical rigid body dynamics method initially analyzes the motion of a single rigid body, and its application to multiple interconnected rigid bodies is limited. For example, double complex pendulum, gyro and gimbal gyroscopes have several special problems. [1]-[6].

He large-scale motion situation introduces the influence of geometric nonlinearity on various structural forms. In the field of dynamics, the earliest people call it the phenomenon of “powered rigidification”. Modeling analysis and research for such phenomena have improved the accuracy of the model. Significance.

In recent years, many researchers have modeled dynamic equations in terms of introducing geometric nonlinearities. On the other hand, the “one-time model study” established for Kane dynamics is also an extremely valuable research work to improve the accuracy of dynamic models [5].

For the structure of the shape rule, the continuum model can be directly established, and the analytical solution can be directly obtained. In the engineering practice, many structural situations are complicated, various connections and various materials are used, which is simplified into a simple shape and a regular continuum structure. There is a big error. To this end, various approximate solution methods have been developed. The complex continuum problem is reduced to a finite degree of freedom problem by presupposing the vibration shape, and the multi-degree of freedom system theory is applied to solve the problem.

All in all, the Kane dynamics method has its unique characteristics, and its analytical mechanics makes the column dynamics equation more intuitive and understandable. In this paper, the Kane dynamics method is used to model the dynamics of the cantilever plate under arbitrary motion. From the form of the dynamic equation, the "one-time dynamic model" established by the Kane method and the traditional "zero-order dynamic model" will be compared.

2. Kane method kinematics description

In order to make the dynamic equations of the plate model easier to write, the following assumptions are made on the board [6]-[8]:

1) The board material is isotropic;

2) The thickness of the plate is smaller than the other dimensions and the uniform Kirchhoff assumption is true, which also means that the effects of lateral shear and moment of inertia are neglected.;

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Such a hypothesis simplifies the derivation process, so that the focus changes due to the large-scale motion can be visually reflected in the equation..

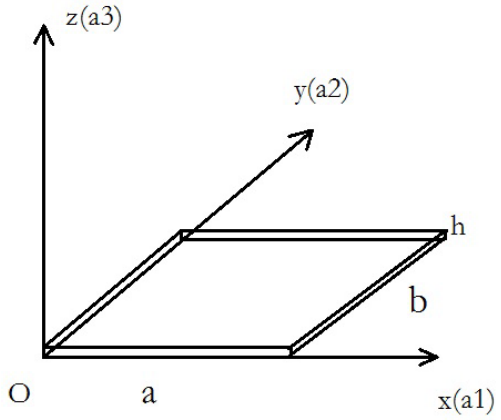

Figure 1 Rectangular plate model and coordinate representation

In this paper, the dynamic equation based on the Kane method is established for the rectangular plate for any large-scale motion. Then the boundary conditions are introduced to solve the corresponding modal function of the cantilever plate vibration, and the cantilever plate dynamic equation of the specific motion can be obtained. . In the analysis of this paper, for the simple analysis, the derivation and calculation analysis of the Kane dynamic equations are carried out by using the plate structure for fixed axis rotation[9]-[12].

In Figure 1, O-xyz is a conjoined coordinate system, and the base vectors are (a_1, a_2, a_3). respectively. The size of the plate is a × b × h. It is assumed that any mass point above and below the deformation of the plate moves from P to P_0, and the deformation displacement is $u\ (u_1, u_2, u_3)$.

Then the velocity vector of point P in the inertial coordinate system can be expressed as:

$$v_p = v_0 + w_a \times (\rho + u) + v_{pa} \tag{1}$$

In the equation:v_o---- The velocity vector of the connected coordinate system in the inertial coordinate system

w_a---- The angular velocity vector of the connected coordinate system in the inertial coordinate system

ρ---- Position vector for P_0 in the conjoined coordinate system

u----Deformation vector for P_0 in the conjoined coordinate system

v_{pa}---- Velocity vector for the relative floating coordinate system of point P

The components of the above formula are:

$$v_0 = v_1 a_1 + v_2 a_2 + v_3 a_3 \tag{2}$$

$$w_a = w_1 a_1 + w_2 a_2 + w_3 a_3 \tag{3}$$

$$v_{pa} = \dot{u}_1 a_1 + \dot{u}_2 a_2 + \dot{u}_3 a_3 \tag{4}$$

$$\rho = x a_1 + y a_2 \tag{5}$$

Substituting the above formula into (1):

$$\begin{aligned} v_p &= [v_1 + \dot{u}_1 + w_2 u_3 - w_3 (y + u_2)] a_1 \\ &+ [v_2 + \dot{u}_2 - w_1 u_3 + w_3 (x + u_1)] a_2 \\ &+ [v_3 + \dot{u}_3 + w_1 (y + u_2) - w_2 (x + u_1)] a_3 \end{aligned} \tag{6}$$

The acceleration vector of P in the inertial coordinate system is obtained for the above formula.:

$$a_p = \frac{\partial v_p}{\partial t} + \omega_a \times v_p = C_1 a_1 + C_2 a_2 + C_3 a_3 \tag{7}$$

Where C_1、C_2、C_3 are the coefficients of the simplified representation

$$C_1 = [\dot{v}_1 + w_2 v_3 - w_1 v_2 + \ddot{u}_1 + (w_1 w_3 + \dot{w}_2)u_3 + 2w_2\dot{u}_3 - 2w_3\dot{u}_2 + (w_1 w_2 - \dot{w}_3)(y + u_2) - (w_2^2 + w_3^2)(x + u_1)]$$

$$C_2 = [\dot{v}_2 + w_3 v_1 - w_1 v_3 + \ddot{u}_2 + (w_1 w_2 + \dot{w}_3)(x + u_1) + 2w_3\dot{u}_1 - 2w_1\dot{u}_3 + (w_2 w_3 - \dot{w}_1)u_3 - (w_1^2 + w_3^2)(y + u_2)]$$

$$C_3 = [\dot{v}_3 + w_1 v_2 - w_2 v_1 + \ddot{u}_3 + (w_2 w_3 + \dot{w}_1)(y + u_1) + 2w_1\dot{u}_2 - 2w_2\dot{u}_1 + (w_1 w_3 - \dot{w}_2)(x + u_1) - (w_1^2 + w_2^2)u_3]$$

According to the relevant references, (u_1, u_2, u_3) of the conjoined coordinate system is selected as the premature linear representation of the deformation description, so the variables of the two arc length variables *s,r* and u_3 are described as deformations.

For the convenience of subsequent solution, the s,r,u_3 hypothetical modal method is expressed as follows:

$$s(x, y, t) = \sum_{j=1}^{\mu_1} \phi_{1j}(x, y) q_{1j}(t) \tag{8}$$

$$r(x, y, t) = \sum_{j=1}^{\mu_2} \phi_{2j}(x, y) q_{2j}(t) \tag{9}$$

$$u_3(x, y, t) = \sum_{j=1}^{\mu_3} \phi_{3j}(x, y) q_{3j}(t) \tag{10}$$

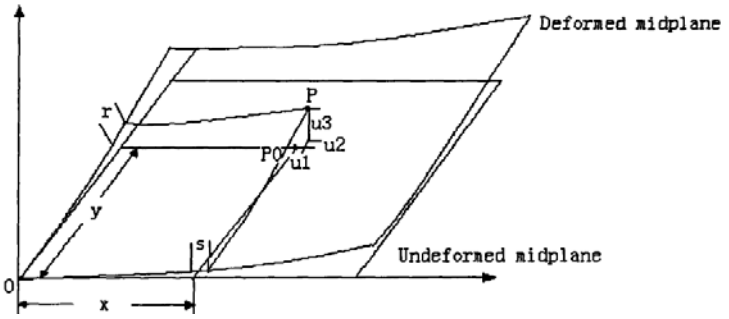

Figure 2 Any point P deformation diagram

In the above formula, $\phi_{1j}(x,y)$、$\phi_{2j}(x,y)$、$\phi_{3j}(x,y)$ are hypothetical modal functions, and any vibration equation solution of the plate satisfying the boundary condition can be used as a modal function. $q_{1j}(t)$、$q_{2j}(t)$、$q_{3j}(t)$ are the selected generalized coordinates, and μ_1、μ_2、μ_3 are the selected generalized coordinates and also the number of truncated modes.

In order to adopt the selected generalized coordinates s,r,u_3, it should be applied to v_p、a_p instead of u_1、u_2.

Applying the two-term expansion theorem, the above equation can be simplified to:

$$s = u_1 + \frac{1}{2}\int_0^x (\frac{\partial u_3}{\partial \xi})^2 d\xi \tag{11}$$

$$r = u_2 + \frac{1}{2}\int_0^y (\frac{\partial u_3}{\partial \eta})^2 d\eta \tag{12}$$

By the above two formulas, u_1、u_2、$\dot{u}_1$、$\dot{u}_2$ can be replaced by(6),(7). As s、r、$\dot{s}$、$\dot{r}$. It can be used as kinematics to describe parameters by s、r、u_3.

By the simultaneous solution of equations(6) to (10). The partial derivative of v_p with respect to the generalized coordinate $\dot{q}_r$ can be obtained as:

$$\frac{\partial v_p}{\partial \dot{q}_r} = \left[\phi_{1i} - \sum_{j=1}^{\mu} \int_0^x \phi_{3i,\xi} \phi_{3j,\xi} d\xi q_i \right] a_1 + \left[\phi_{2i} - \sum_{j=1}^{\mu} \int_0^y \phi_{3i,\eta} \phi_{3j,\eta} d\eta q_j \right] a_2 + \left[\phi_{3i} \right] a_3 \tag{13}$$

$$(r = 1,2,\ldots \mu . \quad \mu = \mu_1 + \mu_2 + \mu_3)$$

In the above formula:ϕ_{1i}、ϕ_{2i}、ϕ_{3i} respectively satisfy the following conditions:

$$\phi_{1i} = 0(i > \mu_1)$$

$$\phi_{2i} = 0(i < \mu_1, i > \mu_1 + \mu_2)$$

$$\phi_{3i} = 0(i < \mu_1 + \mu_2, i > \mu_1 + \mu_2 + \mu_3)$$

The above formula is used to calculate the general inertial force in the dynamic equation.

3. Kinetic equations of the Kane method

3.1 Generalized main power

The strain energy of the plate [14] can be expressed as:

$$U = U_i + U_b \tag{14}$$

Where U_i and U_b represent the deformation potential energy of the plate in the plane and the bending deformation potential energy of the plate, respectively, which can be expressed as follows:

$$U_i = \frac{1}{2} \int_0^b \int_0^a \left[\beta_1 \left\{ \left(\frac{\partial s}{\partial x} \right)^2 + \left(\frac{\partial r}{\partial y} \right)^2 + 2v \frac{\partial s}{\partial x} \frac{\partial r}{\partial y} \right\} + \beta_2 \left(\frac{\partial s}{\partial y} + \frac{\partial r}{\partial x} \right)^2 \right] dxdy \tag{15}$$

$$U_b = \frac{1}{2} \int_0^b \int_0^a \left[\beta_3 \left\{ \left(\frac{\partial^2 u_3}{\partial x^2} \right)^2 + \left(\frac{\partial^2 u_3}{\partial y^2} \right)^2 + 2v \frac{\partial^2 u_3}{\partial x^2} \frac{\partial^2 u_3}{\partial y^2} \right\} + 2\beta_3 (1-v) \left(\frac{\partial^2 u_3}{\partial x \partial y} \right)^2 \right] dxdy \tag{16}$$

In the above formula, the upper limits *a,b* of the integral represent the length and width of the plate, respectively, and β_1、β_2、β_3 represent respectively.:

$$\beta_1 = Eh / ((1 - v^2)) \tag{17}$$

$$\beta = Gh \tag{18}$$

$$\beta_3 = (Eh^3) / (12(1 - v^2)) \tag{19}$$

And *E, G, v, h* represent the elastic modulus, shear modulus, Poisson's ratio and thickness of the plate, respectively.

In the above formula, taking *s* and *r* represents the actual amount of deformation, so the deformation in the plate can represent more accurate actual deformation energy.

Combining the above two formulas, the generalized main power can be obtained by applying the following formula:

$$F_i = -\frac{\partial U}{\partial q_i} \quad (i = 1,2,\ldots,\mu) \tag{20}$$

The generalized principal power component form is::

$$F_i = -\begin{cases} \sum_{j=1}^{\mu_1} K_{ij}^{S1} q_{1j} + \sum_{j=1}^{\mu_2} K_{ij}^{S2} q_{2j} (r = i, i = 1,2,...,\mu_1) \\ \sum_{k=1}^{\mu_2} K_{jk}^{S3} q_{2k} + \sum_{k=1}^{\mu_1} K_{jk}^{S4} q_{1k} (r = \mu_1 + j,\ j = 1,2,...,\mu_2) \\ \sum_{i=1}^{\mu_3} K_{ki}^{B} q_{3i} (r = \mu_1 + \mu_2 + k, k = 1,2,...,\mu_3) \end{cases} \tag{21}$$

In the above formula:

$$K_{ij}^{S1} = \int_0^b \int_0^a (\beta_1 \phi_{1i,x} \phi_{1j,x} + \beta_2 \phi_{1i,y} \phi_{1j,y}) dxdy$$

$$K_{ij}^{S2} = \int_0^b \int_0^a (\beta_1 \nu \phi_{1i,x} \phi_{2j,y} + \beta_2 \phi_{1i,y} x) dxdy$$

$$K_{ij}^{S3} = \int_0^b \int_0^a (\beta_1 \phi_{2i,y} \phi_{2j,y} + \beta_2 \phi_{2i,x} \phi_{2j,x}) dxdy$$

$$K_{ij}^{S4} = \int_0^b \int_0^a (\beta_1 \nu \phi_{2i,y} \phi_{1j,x} + \beta_2 \phi_{2i,x} \phi_{2j,y}) dxdy$$

$$K_{ij}^{B} = \int_0^b \int_0^a \beta_3 [\phi_{3i,xx} \phi_{3j,xx} + \phi_{3i,yy} \phi_{3j,yy} + \nu \phi_{3i,xx} \phi_{3j,yy} + \nu \phi_{3i,yy} \phi_{3j,xx} + 2(1-\nu) \phi_{3i,xy} \phi_{3j,xy}] dxdy$$

3.2 Generalized inertial force

Generalized inertial force can be obtained by definition:

$$F_i^* = -\int_0^b \int_0^a \rho \frac{\partial v_p}{\partial \dot{q}_i} \bullet a_p dxdy \quad (i = 1,2,\ldots,\mu,\ \ \mu = \mu_1 + \mu_2 + \mu_3) \tag{22}$$

Where ρ is the areal density of the plate,$\dot{q}_\iota$ is the derivative of the generalized coordinate versus time, and v_p, a_p is the P-point substitution obtained in the previous paragraph, then:

$$\begin{aligned} -F_i^* &= (\dot{v}_1 + w_2 v_3 - w_3 v_2) Z_{1i} + (w_1 w_2 - \dot{w}_3) Y_{1i} - (w_2^2 + w_3^2) X_{1i} + \\ &\sum_{j=1}^{\mu_1} W_{ij}^{11} \ddot{q}_{1j} + 2 w_2 \sum_{j=1}^{\mu_3} W_{ij}^{13} \dot{q}_{3j} + (w_1 w_3 + \dot{w}_2) \sum_{j=1}^{\mu_3} W_{ij}^{13} q_{3j} + \\ &(w_1 w_2 - \dot{w}_3) \sum_{j=1}^{\mu_2} W_{ij}^{12} q_{2j} - 2 w_3 \sum_{j=1}^{\mu_3} W_{ij}^{12} \dot{q}_{2j} - (w_2^2 + w_3^2) \sum_{j=1}^{\mu_1} W_{ij}^{11} q_{1j} \\ &(r = i, i = 1,2,...,\mu_1) \end{aligned} \tag{23}$$

$$\begin{aligned} -F_{\mu_1+j}^* &= (\dot{v}_2 + w_3 v_1 - w_1 v_3) Z_{2j} + (w_1 w_2 - \dot{w}_3) X_{2j} - (w_2^2 + w_3^2) X_{2j} + \\ &\sum_{j=1}^{\mu_2} W_{ij}^{22} \ddot{q}_{2j} + 2 w_3 \sum_{i=1}^{\mu_1} W_{ji}^{21} \dot{q}_{1i} + (w_1 w_2 + \dot{w}_3) \sum_{i=1}^{\mu_1} W_{ji}^{21} q_{1i} + \\ &(w_2 w_3 - \dot{w}_1) \sum_{k=1}^{\mu_3} W_{jk}^{23} q_{3k} - 2 w_1 \sum_{k=1}^{\mu_3} W_{jk}^{23} \dot{q}_{3k} - (w_1^2 + w_3^2) \sum_{k=1}^{\mu_1} W_{jk}^{22} q_{2k} \\ &(r = \mu_1 + j,\ j = 1,2,...,\mu_2) \end{aligned} \tag{24}$$

$$\begin{aligned} -F_{\mu_1+\mu_2+j}^* &= (\dot{v}_3 + w_1 v_2 - w_2 v_1) Z_{3k} + (w_2 w_3 + \dot{w}_1) Y_{3k} - (w_1 w_3 + \dot{w}_2) X_{3k} + \\ &\sum_{i=1}^{\mu_3} W_{ki}^{33} \ddot{q}_{2i} + 2 w_1 \sum_{i=1}^{\mu_1} W_{ki}^{32} \dot{q}_{2i} + (w_2 w_3 + \dot{w}_1) \sum_{i=1}^{\mu_2} W_{ki}^{32} q_{2i} + \\ &(w_1 w_3 - \dot{w}_2) \sum_{i=1}^{\mu_1} W_{ki}^{31} q_{1i} - 2 w_2 \sum_{k=1}^{\mu_1} W_{jk}^{31} \dot{q}_{1i} - (w_1^2 + w_2^2) \sum_{k=1}^{\mu_3} W_{ki}^{33} q_{3k} + \\ &(w_2^2 + w_3^2) \sum_{i=1}^{\mu_3} K dx_{ki}^2 q_{3i} - (\dot{v}_1 + w_2 v_3 - w_3 v_2) \sum_{i=1}^{\mu_3} K dx_{ki}^0 q_{3i} + \\ &(w_1^2 + w_3^2) \sum_{i=1}^{\mu_3} K dy_{ki}^2 q_{3i} - (\dot{v}_2 + w_3 v_1 - w_1 v_3) \sum_{i=1}^{\mu_3} K dx_{ki}^0 q_{3i} - \\ &(w_1 w_3 - \dot{w}_2) \sum_{i=1}^{\mu_3} K dy_{ki}^1 q_{3i} + (w_1 w_2 + \dot{w}_3) \sum_{i=1}^{\mu_3} K dx_{ki}^1 q_{3i} \\ &(r = \mu_1 + \mu_2 + k, k = 1,2,...,\mu_3) \end{aligned} \tag{25}$$

In the above formula:

$$W_{ij}^{lm}=\int_0^b\int_0^a \rho\phi_{li}\phi_{mj}dxdy, X_{mi}=\int_0^b\int_0^a \rho x\phi_{mi}dxdy$$

$$Y_{mi}=\int_0^b\int_0^a \rho y\phi_{mi}dxdy, Z_{mi}=\int_0^b\int_0^a \rho\phi_{mi}dxdy$$

$$Kdx_{ij}^0=\int_0^b\int_0^a \rho(a-x)\phi_{3i,x}\phi_{3j,x}dxdy, Kdx_{ij}^1=\int_0^b\int_0^a \rho[x(b-y)\phi_{3i,y}\phi_{3j,y}]dxdy$$

$$Kdx_{ij}^2=\int_0^b\int_0^a \frac{1}{2}\rho(a^2-x^2)\phi_{3i,x}\phi_{3j,x}dxdy, Kdy_{ij}^0=\int_0^b\int_0^a \rho(b-y)\phi_{3i,y}\phi_{3j,y}dxdy$$

$$Kdy_{ij}^1=\int_0^b\int_0^a \rho[y(a-x)\phi_{3i,x}\phi_{3j,x}]dxdy, Kdy_{ij}^2=\int_0^b\int_0^a \frac{1}{2}\rho(b^2-y^2)\phi_{3i,y}\phi_{3j,y}dxdy$$

3.3 Kane dynamic equation

Substituting the generalized primary and general inertial forces obtained in the first two sections into the Kane dynamic equation is simplified:

$$\begin{aligned}&\sum_{j=1}^{\mu_1}W_{ij}^{11}\ddot{q}_{1j}-2w_3\sum_{j=1}^{\mu_2}W_{ij}^{12}\dot{q}_{2j}+2w_2\sum_{j=1}^{\mu_3}W_{ij}^{13}\dot{q}_{3j}-(w_2^2+w_3^2)\sum_{j=1}^{\mu_1}W_{ij}^{11}q_{1j}\\&+(w_1w_2-\dot{w}_3)\sum_{j=1}^{\mu_2}W_{ij}^{12}q_{2j}+(w_1w_3+\dot{w}_2)\sum_{j=1}^{\mu_3}W_{ij}^{13}q_{3j}+\sum_{j=1}^{\mu_1}K_{ij}^{S1}q_{1j}\\&+\sum_{j=1}^{\mu_2}K_{ij}^{S2}q_{2j}=(w_2^2+w_3^2)X_{1i}-(w_1w_2-\dot{w}_3)Y_{1i}-(\dot{v}_1+w_2v_3-w_3v_2)Z_{1i}\end{aligned}\tag{26}$$

$$\begin{aligned}&\sum_{i=1}^{\mu_2}W_{ji}^{22}\ddot{q}_{2i}+2w_3\sum_{i=1}^{\mu_1}W_{ji}^{21}\dot{q}_{1i}-2w_1\sum_{k=1}^{\mu_3}W_{jk}^{23}\dot{q}_{3k}+(w_1w_2+\dot{w}_3)\sum_{i=1}^{\mu_1}W_{ji}^{21}q_{1i}\\&+(w_2w_3-\dot{w}_1)\sum_{k=1}^{\mu_3}W_{jk}^{23}q_{3k}-(w_1^2+w_3^2)\sum_{k=1}^{\mu_2}W_{jk}^{22}q_{2k}+\sum_{k=1}^{\mu_2}K_{jk}^{S3}q_{2k}\\&+\sum_{k=1}^{\mu_1}K_{jk}^{S4}q_{1k}=-(w_1w_2+\dot{w}_3)X_{2j}+(w_1^2+w_3^2)Y_{2j}-(\dot{v}_2+w_3v_1-w_1v_2)Z_{2j}\end{aligned}\tag{27}$$

$$\begin{aligned}&\sum_{i=1}^{\mu_3}W_{ki}^{33}\ddot{q}_{3i}+2w_1\sum_{i=1}^{\mu_3}W_{ki}^{32}\dot{q}_{2i}-2w_2\sum_{k=1}^{\mu_1}W_{ki}^{31}\dot{q}_{1i}+(w_2w_3+\dot{w}_1)\sum_{i=1}^{\mu_2}W_{ki}^{32}q_{2i}\\&+(w_1w_3+\dot{w}_2)\sum_{i=1}^{\mu_1}W_{ki}^{31}q_{1i}-(w_1^2+w_2^2)\sum_{k=1}^{\mu_3}W_{ki}^{33}q_{2i}+(w_2^2+w_3^2)\sum_{k=1}^{\mu_3}Kdx_{ki}^2q_{3i}\\&-(\dot{v}_1+w_2v_3-w_3v_2)\sum_{i=1}^{\mu_3}Kdx_{ki}^0q_{3i}-(w_1w_2-\dot{w}_3)\sum_{i=1}^{\mu_3}Kdx_{ki}^1q_{3i}+(w_1^2+w_3^2)\\&\sum_{i=1}^{\mu_3}Kdy_{ki}^2q_{3i}-(\dot{v}_2+w_3v_1-w_1v_3)\sum_{i=1}^{\mu_3}Kdy_{ki}^0q_{3i}+(w_1w_2+\dot{w}_3)\sum_{i=1}^{\mu_3}Kdy_{ki}^1q_{1i}\\&=-(w_1w_3-\dot{w}_2)X_{3k}-(w_2w_3+\dot{w}_1)Y_{3k}-(\dot{v}_3+w_1v_2-w_2v_1)Z_{3k}\end{aligned}\tag{28}$$

Similar to the dynamic equation of the beam model, K_{ki}^{dX} and other terms represent the dynamic stiffness term, and its contribution to the system is related to the speed and angular velocity of the system. If you ignore this idea, you may get a zero-order approximate kinetic model, which may lead to a completely wrong conclusion when the speed is high.

4. Conclusion

In this thesis, the Kane method is used to dynamically model the spacecraft and the flexible attachment. The selection of the coordinate description method is essentially the selection of the generalized rate, which is different from the traditional generalized coordinates, but has a certain correspondence. . Each of the literature on the Kane method indicates how to reasonably select a generalized rate, which needs to be determined based on the experience of the researcher.

The selection of the generalized rate adopted in the paper is the selection method of the reference correlation model. On this basis, the dynamic equation corresponding to the Kane method is derived. The cantilever plate of any motion in space was modeled. In the modeling process, combined with the hypothetical modal method, through the modeling and analysis of the vibration characteristics of the flexible attachment, combined with the overall motion of the spacecraft, the dynamic model of the flexible plate attachment in the simple motion mode is obtained.

The analysis of the dynamic equations of the established model shows that the traditional modeling method, based on the simplified coupling relationship between the motion variables, lacks the dynamics of the dynamic equations compared with the Kane method. Coupling terms, which affect the accuracy of modeling calculations.

References

[1] Zhai Bao, Yu Ting, Wang Guoping, et al. Progress in multibody system dynamics research[J]. Journal of Vibration and Shock, 2011, 30(7): 178-187.*(in Chinese)*

[2] Kane T R, Ryan R, Banerjee A K. Dynamics of a cantilever beam attached to a moving base[J]. Journal of Guidance, Control, and Dynamics, 1987, 10(2): 139-151.

[3] Kai En, T.R. Spacecraft Dynamics [M]. Science Press, 1988. *(in Chinese)*

[4] Xu Hongwei, Ma Xingrui. Discussion on dynamics of two-arm coordination and restraint system[J]. Journal of Vibration Engineering, 1994(1): 1-8. *(in Chinese)*

[5] Liu Yanzhu. Higher Dynamics [M]. Higher Education Press, 2016. *(in Chinese)*

[6] Zhang Yiyao. Research on rigid-flexible coupled dynamics modeling and dynamic characteristics of flexible spacecraft [D]. Harbin Institute of Technology, 2015 *(in Chinese)*

[7] Li Cuichun. Research on rigid-flexible coupled dynamics modeling and attitude control technology for flexible spacecraft [D]. Beijing Institute of Technology, 2014. *(in Chinese)*

[8] Luo Wen. Rigid-flexible coupled dynamics modeling of solar wing satellites [D]. Harbin Institute of Technology, 2015. *(in Chinese)*

[9] Wallrapp O. Linearized Flexible Multibody Dynamics Including Geometric Stiffening Effects∗[J]. Journal of Structural Mechanics, 1991, 19(3): 385-409.

[10]Likins P W. Finite element appendage equations for hybrid coordinate dynamic analysis[J]. International Journal of Solids and Structures, 1972, 8(5): 709-731.

[11]Wang J T, Huston R L. Kane's equations with undetermined multipliers—application to constrained multibody systems[J]. Journal of applied mechanics, 1987, 54(2): 424-429.

[12]Huston R L. Dynamics of constrained multibody systems[J]. Journal of Applied Mechanics, 1984, 51: 899.

[13]Yoo H H, Ryan R R, Scott R A. Dynamics of flexible beams undergoing overall motions[J]. Journal of Sound and vibration, 1995, 181(2): 261-278.

[14]Yoo H H, CHUNG J. Dynamics of rectangular plates undergoing prescribed overall motion[J]. Journal of Sound and Vibration, 2001, 239(1): 123-137.