Fingerprint Identification Based on Wavelet Texture Features of Matlab

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Abstract: At first, this paper introduces the basic theory of wavelet analysis, such as continuous wavelet transform, discrete wavelet transform and wavelet packet analysis. Wavelet transform has the characteristics of time-frequency localization, so it can not only provide more accurate time domain location for fingerprints, but also provide more accurate frequency domain location. The fingerprints after wavelet transform have the characteristics of spectrum division, direction selection, multi-resolution analysis and natural tower data structure. The basic methods of fingerprint compression, fingerprint denoising, fingerprint fusion, fingerprint decomposition and fingerprint enhancement in MATLAB language environment are discussed on the basis of these characteristics of wavelet transform.

1. Introduction

Wavelet analysis was born in the 1980s and is considered as a new stage of harmonic analysis, that is, the development of modern Fourier analysis. Many high-tech are based on mathematics and wavelet analysis is known as "mathematical microscope", which determines its important position in the field of high-tech research. Now its applications in pattern identification, fingerprint identification, speech processing, fault diagnosis, geophysical exploration, fractal theory, aerodynamics and fluid mechanics have been extensively and deeply studied, and it has been applied even in social sciences such as finance, securities, stocks and so on.

The signal is completely spread out in the frequency domain and does not contain any time-frequency information in traditional Fourier analysis, which is very appropriate for some applications because the frequency information of the signal is very important to it. And the discarded time-domain information may be very important for some applications, so people have popularized Fourier analysis and proposed many signal analysis methods which can represent time-domain and frequency-domain information, such as short-time Fourier transform, Gabor transform, time-frequency analysis, wavelet transform, etc. Among them, short-time Fourier transform is the first attempt to introduce time-domain information based on Fourier analysis. Its basic assumption is that the signal is stable in a certain time window, so the local frequency-domain information can be obtained after expanding the signal into the frequency-domain in each time window through dividing the time window. However, its time-domain discrimination can only depend on the time window of constant size which is still too big for some transient signals. In other words, short-term Fourier analysis can only be performed at one resolution. So it is not accurate enough for many applications, and there are many defects.

Wavelet analysis overcomes the defects of short-time Fourier transform in single resolution with the characteristics of multi-resolution analysis. It has the ability to represent the local information of signals in both time and frequency domains, its time window and frequency window can be dynamically adjusted according to the specific shape of signals. In general, lower time resolution can be adopted in low frequency part (signal is stable). For the resolution of improving frequency, lower frequency resolution can be used in high-frequency cases (frequency changes little) to exchange for accurate time positioning.

This paper introduces the basic theory of wavelet transform and some frequently used wavelet
functions. Their main properties include compact support length, filter length, symmetry, vanishing moment and so on, which have given a brief explanation. Then the application of wavelet analysis in fingerprint identification is studied, including fingerprint compression, fingerprint denoising, fingerprint fusion, fingerprint decomposition, fingerprint enhancement, etc.

2. Basic Theory of Wavelet Analysis

2.1 Continuous Wavelet Transform

Definition: Let \( \psi(t) \in L^2(\mathbb{R}) \), its Fourier transform be \( \hat{\psi}(\omega) \), when \( \hat{\psi}(\omega) \) is satisfied with enable condition (complete reconstruction condition or identical resolution condition),

\[
C_\psi = \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty
\]

we call \( \psi(t) \) is a basic or mother wavelet. This is obtained by dilation and translation of the generating function:

\[
\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right) \quad a, b \in \mathbb{R}; a \neq 0
\]

It is called a wavelet sequence. Among them, 'a' is the dilation factor and 'b' is the translation factor. For any function, its continuous wavelet transform is:

\[
W_f(a,b) = \langle f, \psi_{a,b} \rangle = |a|^{-1/2} \int_{-\infty}^{\infty} f(t) \overline{\psi\left(\frac{t-b}{a}\right)} dt
\]

Its reconstruction formula (inverse transformation) is:

\[
f(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{|a|^2} W_f(a,b) \psi\left(\frac{t-b}{a}\right) dadb
\]

Because the wavelet \( \psi_{a,b}(t) \) generated by the basic wavelet \( \psi(t) \) acts as an observation window for the signal being analyzed in the wavelet transform, \( \psi(t) \) should also satisfy the constraint condition of the general function.

\[
\int_{-\infty}^{\infty} |\psi(t)| dt \langle \infty
\]

So \( \hat{\psi}(\omega) \) is a continuous function. This means that \( \hat{\psi}(\omega) \) must be equal to '0' at the origin in order to satisfy the complete reconstruction condition.

That is

\[
\hat{\psi}(0) = \int_{-\infty}^{\infty} \psi(t) dt = 0
\]

In order to make the realization of signal reconstruction numerically stable, besides dealing with the condition of complete reconstruction, the Fourier transform of the wavelet is required to satisfy the following stability conditions:

\[
A \leq \sum_{-\infty}^{\infty} |\hat{\psi}(2^{-j}\omega)|^2 \leq B
\]

In formula, \( \langle A \leq B \langle \infty \).
2.2 Discrete Wavelet Transform

In practical application, especially in computer, continuous wavelet must be discretized. Therefore, it is necessary to discuss the discretization of continuous wavelet $\psi_{a,b}(t)$ and continuous wavelet transform $W_f(a,b)$. It should be emphasized that, this discretization is for the continuous scale parameter 'a' and the continuous translation parameter 'b', not for the time variable 't'. This is different from the time discretization we used to have. In continuous wavelet, the function is considered:

$$\psi_{a,b}(t) = |a|^{-1/2} \psi \left( \frac{t-b}{a} \right)$$

In here, $b \in R$, $a \in R^+$, and $a \neq 0$, $\psi$ are permissible. For convenience, the total limit 'a' only be positive number in discretization, so the compatibility condition becomes

$$C_\psi = \int_0^\infty \frac{\hat{\psi}(\omega)}{\omega} d\omega < \infty$$

Generally, the mesoscale parameter 'a' of continuous wavelet transform and the translation parameter 'b' of discrete formulas are taken as respectively $a_0 a^j$, $b = ka^j b_0$, in here $j \in Z$. The extended step $a_0 \neq 1$ is a fixed value, and for convenience it is always assumed that $a_0 > 1$ (due to 'm' is either positive or negative number, this assumption does not matter). So the corresponding discrete wavelet function $\psi_{j,k}(t)$ can be written as:

$$\psi_{j,k}(t) = a_0^{-j/2} \psi \left( \frac{t-k a_0^j b_0}{a_0^j} \right) = a_0^{-j/2} \psi(a_0^{-j} t - kb_0)$$

And the discretized wavelet transform coefficients can be expressed as

$$C_{j,k} = \int_{-\infty}^{\infty} f(t) \psi^*_{j,k}(t) dt = < f, \psi_{j,k} >$$

Its reconstruction formula is as follows:

$$f(t) = C \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} C_{j,k} \psi_{j,k}(t)$$

'C' is a signal independent constant. However, how to choose $a_0$ and $b_0$ to ensure the accuracy of reconstructed signal? Obviously, grid points should be as dense as possible (that is, $a_0$ and $b_0$ are as small as possible), because the more sparse grid points are, the less wavelet function $\psi_{j,k}(t)$ and discrete wavelet coefficients $C_{j,k}$ are used, and the lower the accuracy of signal reconstruction is.

2.3 Wavelet Packet Analysis

Short-time Fourier transform divides the frequency band of the signal into linear equal intervals. Multiresolution analysis can effectively decompose the signal in time-frequency domain, but its frequency resolution is poor in high frequency band and its time resolution is poor in low frequency band because its scale varies according to binary system. Which is the frequency band of the signal is divided exponentially at equal intervals (with equal 'Q' structure). Wavelet packet analysis can provide a more precise method for signal analysis. It divides the frequency band into multiple levels, further decomposes the high frequency part which has no subdivision in multi-resolution analysis,
and adaptively chooses the corresponding frequency band according to the characteristics of the analyzed signal, so as to match the frequency spectrum of the signal and improve the time-frequency resolution. Therefore, the wavelet packet has a wider application value.

As for the understanding of wavelet packet analysis, we illustrate it with a three-level decomposition. Its wavelet packet decomposition tree is shown in the figure:

In Figure 1, 'A' represents low frequency, 'D' represents high frequency, and the ordinal number at the end represents the layer tree (it also be called scale number) of wavelet decomposition. The formula of decomposition:

\[
S = AAA3 + DAA3 + ADA3 + DDA3 + AAD3 + DAA3 + ADD3 + DDD3
\]

3. Application of Wavelet Analysis in Fingerprint Fusion

Fingerprint fusion refers to combine two or more fingerprints of the same object into one fingerprint so that it can be understood more easily than any original fingerprint. This technology can be applied to multi-spectral fingerprint understanding, medical fingerprint identification and so on. On these occasions, fingerprints of the same object are usually obtained by different imaging mechanisms.

The two fingerprints of woman. mat and wbarb. mat in the above example are fused by two-dimensional wavelet analysis. List of procedures:

The outputs are as follows:

Figure 2 Wavelet analysis applied in fingerprint fusion
4. Summary

This paper mainly combines the basic concepts and principles of wavelet transform to discuss the application of wavelet in fingerprint identification in detail, and illustrate the application with MATLAB programming language.

The first key point is to understand the theory and method of wavelet transform and wavelet analysis. This paper mainly studies the basic theories of continuous wavelet transform, discrete wavelet transform and wavelet packet analysis, and introduces some frequently used wavelet bases. The second key point is to study the application of wavelet analysis and wavelet transform in fingerprint recognition.

References