Research on the Spare Parts Consumption Rule Based on Prior Distributions

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Abstract: Prediction models of spare parts consumption based on prior distributions were developed. Based on the analysis of the basic methods of acquiring prior distributions and the bayesian theory, through comprehensive utilization of the prior distribution and sample data, models of spare parts consumption that change with time based on prior distributions were developed. Then the combined methods of sample data were presented and the models of spare parts consumption in many pieces of equipment were developed.

1. Introduction

In the process of equipment design, production, test, finalization and the use of equipment units, there is a series of information closely related to spare parts consumption. The prior distribution of spare parts consumption can be obtained by using these information, and the accuracy of spare parts consumption prediction can be improved by using the existing sample data to correct the prior distribution. The method of spare parts consumption prediction based on prior distribution makes full use of all kinds of information and enlarges the information quantity of spare parts consumption rule evaluation. Therefore, in the evaluation of spare parts consumption law, attention should be paid to the accumulation, collection, mining and processing of various information to form a prior distribution and apply it to statistical inference. Only in this way can the accuracy and reliability of spare parts consumption law evaluation be guaranteed.

The acquisition methods of prior distribution are various, mainly through the following ways: (1) acquiring prior distribution based on the consumption data accumulated from design to finalization; (2) using the consumption data accumulated during the actual use and maintenance of equipment to obtain prior distribution; (3) preparing consumption data generated by modeling and simulation to obtain prior distribution; (4) obtaining prior distribution based on expert experience knowledge.

In this paper, bayesian theory is applied to the prediction of the consumption distribution of constant-consumption spare parts. This method can improve the prediction accuracy and further realize the accurate guarantee of constant-consumption spare parts.

2. Distribution Function of Consumption of Constant-Consumption Spare Parts

(1) Uniform distribution

If the probabilistic density of spare parts consumed by a unit in equipment in $[\tau_1, \tau_2]$ is satisfied

$$f(t) = \frac{1}{\tau_2 - \tau_1}, \tau_1 \le t \le \tau_2 \tag{1}$$

Then the consumption of the constant-consumption spare parts obeys uniform distribution, as shown in Figure 1.

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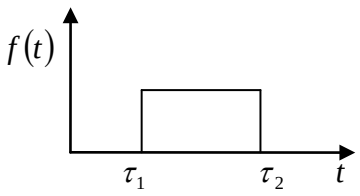

Figure 1 Uniform Distribution Diagram

Discussion:

Mathematical expectation:

$$E(T)=\int_{\tau_1}^{\tau_2} tf(t)\mathrm{d}t=\int_{\tau_1}^{\tau_2} t\frac{1}{\tau_2-\tau_1}\mathrm{d}t=\frac{\tau_2+\tau_1}{2}$$

Consumption Probability of Constant-Consumption Spare Parts in $[T_1,T_2]\subseteq[\tau_1,\tau_2]$ is

$$p=\int_{T_1}^{T_2} f(t)\mathrm{d}t=\int_{T_1}^{T_2}\frac{1}{\tau_2-\tau_1}\mathrm{d}t=\frac{T_2-T_1}{\tau_2-\tau_1}$$

(2) Triangular distribution

If the probabilistic density of spare parts consumed by a unit in equipment in $[\tau_1,\tau_2]$ is satisfied

$$f(t)=\begin{cases}a_1(t-\tau_1) & \tau_1\le t\le\tau_0\\ a_2(\tau_2-t) & \tau_0<t\le\tau_2\end{cases} \tag{2}$$

among them:

$$a_1=\frac{2}{(\tau_2-\tau_1)(\tau_0-\tau_1)},a_2=\frac{2}{(\tau_2-\tau_1)(\tau_2-\tau_0)}$$

Then the consumption of the constant-consumption spare parts obeys the triangular distribution, as shown in Figure 2.

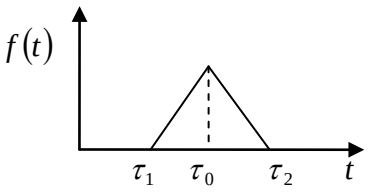

Figure 2 Diagram of triangular distribution

Discussion:

Mathematical expectation:

$$E(T)=\int_{\tau_1}^{\tau_2} tf(t)\mathrm{d}t=\frac{a_1}{3}(t_0^3-t_1^3)-\frac{a_1}{2}t_1(t_0^2-t_1^2)+\frac{a_2}{2}t_2(t_2^2-t_0^2)-\frac{a_2}{3}(t_2^3-t_0^3)$$

Consumption Probability of Constant-Consumption Spare Parts in $[\tau_1,T_2]$ can be divided into three types:

①When $T_2<\tau_0$,

$$p = \int_{\tau_1}^{T_2} f(t)\mathrm{d}t = \frac{a_1}{2}(T_2 - \tau_1)^2$$

②When $\tau_0 < T_2 < \tau_2$,

$$p = \int_{\tau_1}^{T_2} f(t)\mathrm{d}t = \frac{a_1}{2}(\tau_0 - \tau_1)^2 + \frac{a_2}{2}(\tau_0^2 - T_2^2 + 2\tau_2 T_2 - 2\tau_2\tau_0)$$

③When $T_2 > \tau_2$,

$$p = \int_{\tau_1}^{T_2} f(t)\mathrm{d}t = \frac{a_1}{2}(\tau_0 - \tau_1)^2 + \frac{a_2}{2}(\tau_2 - \tau_0)^2 = 1$$

3. Prediction Model Based on Prior Distribution

AtUnder continuous conditions, the posterior distribution probability density function of constant spare parts consumption is

$$\pi(t|D) = \frac{f(t)\pi(t)}{\int_a^b f(t)\pi(t)\mathrm{d}t}$$

Among them, the upper limit value and the lower limit value of the range of values are chosen respectively.

Because of the fixed value, the posterior distribution probability density function of spare parts consumption can be expressed as

$$\pi(t|D) \propto f(t)\pi(t) \tag{3}$$

Formula (3) shows that the posterior distribution probability density function of spare parts consumption is proportional to the product of the prior distribution probability density function and the sample distribution probability density function.

Sample distribution and prior distribution in general Bayesian formula are continuous distribution, and there is only one space. When forecasting the consumption of constant-consumption spare parts, we sometimes encounter a priori distribution and sample distribution as piecewise continuous functions, such as triangular distribution and trapezoidal distribution, so we need to extend Bayesian formula to multiple spaces to meet the requirements.

(1) A prior distribution consists of two spaces

When the prior distribution consists of two spaces and the sample distribution is only one space, the schematic diagram is shown in Figure 3.

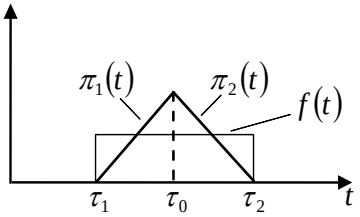

Figure 3 Prior distribution 2 spaces and sample distribution 1 space schematic diagram

From formula (3), the posterior distribution probability density function of spare parts consumption of constant consumption type is obtained.

$$\pi(t|D) \propto \begin{cases} f(t)\pi_1(t) & \tau_1 \le t \le \tau_0 \\ f(t)\pi_2(t) & \tau_0 < t \le \tau_2 \end{cases} \tag{4}$$

(2) Sample distribution consists of two spaces

When the sample distribution consists of two spaces and the prior distribution is only one space, the schematic diagram is shown in Figure 4.

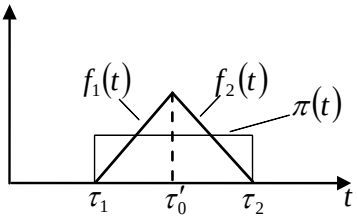

Figure 4 Sample Distribution 2 Spaces and Prior Distribution 1 Spatial Diagram

From formula (3), the posterior distribution probability density function of spare parts consumption of constant consumption type is obtained.

$$\pi(t|D) \propto \begin{cases} f_1(t)\pi(t) & \tau_1 \le t \le \tau_0' \\ f_2(t)\pi(t) & \tau_0' < t \le \tau_2 \end{cases} \tag{5}$$

(3) Prior distribution and sample distribution are composed of two spaces.

When the prior distribution and sample distribution are composed of two spaces, the schematic diagram is shown in Figure 5.

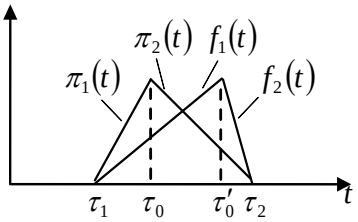

Figure 5 Two spatial schematic maps of prior distribution and sample distribution

When $\tau_0 < \tau_0'$, the probability density function of the posterior distribution of constant spare parts consumption is as follows:

$$\pi(t|D) \propto \begin{cases} f_1(t)\pi_1(t) & \tau_1 \le t \le \tau_0 \\ f_1(t)\pi_2(t) & \tau_0 < t \le \tau_0' \\ f_2(t)\pi_2(t) & \tau_0' \le t \le \tau_2 \end{cases} \tag{6}$$

When $\tau_0 > \tau_0'$, the probability density function of the posterior distribution of constant spare parts consumption is as follows:

$$\pi(t|D) \propto \begin{cases} f_1(t)\pi_1(t) & \tau_1 \le t \le \tau_0' \\ f_2(t)\pi_1(t) & \tau_0' < t \le \tau_0 \\ f_2(t)\pi_2(t) & \tau_0 < t \le \tau_2 \end{cases} \tag{7}$$

Similarly, when the prior distribution or sample distribution is composed of more than three spaces, the probability density function of the posterior distribution can still be obtained by referring to the above methods. The probability density function of the posterior distribution of the consumption of constant-consumption spare parts in this year can be used as the prediction model of the consumption of constant-consumption spare parts in the next year

4. Example Analysis

It is known that the consumption of copper rings in the equipment of a formed unit during the normal training period in 2018. By judging, the type of sample distribution function of the unit consuming copper rings is triangular distribution, and the parameters are $\tau_1 = 4$, $\tau_2 = 10$, $\tau_0 = 6$. At the same time, the prior distribution function type of copper ring consumed by other units is still triangular distribution, and the parameters are $\tau_1 = 4$, $\tau_2 = 10$, $\tau_0' = 7$. The Bayesian theory is used to judge the posterior distribution of copper rings consumed by the unit in 2018, and the consumption of copper rings during the normal training period of equipment in 2019 is predicted.

Solution: According to the question, the probability density function of the sample distribution of copper ring consumption in 2018 is as follows:

$$f(t) = \begin{cases} 0.167(t-4) & 4 \le t \le 6 \\ 0.083(10-t) & 6 < t \le 10 \end{cases}$$

The probability density function of the prior distribution of copper ring consumption in 2018 is as follows:

$$\pi(t) = \begin{cases} 0.111(t-4) & 4 \le t \le 7 \\ 0.111(10-t) & 7 < t \le 10 \end{cases}$$

As $\tau_0 < \tau_0'$, according to formula (7), the probability density function of the posterior distribution of copper ring consumption in 2018 can be obtained as follows:

$$\pi(t|D) \propto \begin{cases} 0.019(t-4)^2 & 4 \le t \le 6 \\ 0.009(10-t)(t-4) & 6 < t \le 7 \\ 0.009(10-t)^2 & 7 < t \le 10 \end{cases}$$

The formula is normalized and the specific probability density function expression of the posterior distribution of copper ring consumption in 2019 is obtained as follows:

$$\pi(t|D) = \begin{cases} 0.232(t-4)^2 & 4 \le t \le 6 \\ 0.377(10-t)(t-4) & 6 < t \le 7 \\ 0.391(10-t)^2 & 7 < t \le 10 \end{cases}$$

The probability density function curves of the prior distribution, sample distribution and posterior distribution of copper ring consumption in 2018 are shown in Figure6 and Figure 7.

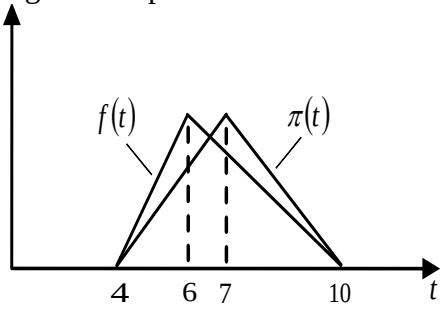

Figure 6 Prior distribution and sample distribution of copper ring consumption

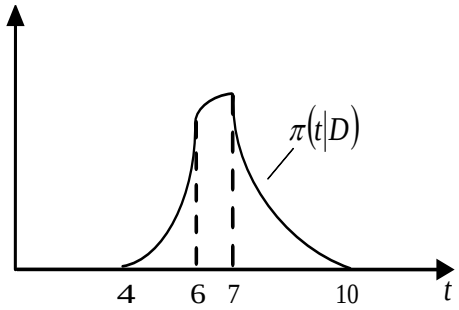

Figure 7 Posterior distribution of copper ring consumption

The posterior distribution of purple copper ring consumption in 2018 is used as the predictive distribution function of purple copper ring consumption during the normal training period in 2019. As can be seen from Figure 7, the consumption of copper rings increases monotonously from April to June in 2019, and the rate of change will be faster; the consumption of copper rings increased monotonously from June to July, but the rate of change gradually will slow down; the consumption of copper rings will decrease monotonously from July to October, and the rate of decline will be faster at the beginning and slower at the later stage. According to the posterior distribution of consumption of copper rings in 2019, the proportion of consumption of copper rings in any period can also be determined, which can ensure the accuracy of copper rings.

After the actual consumption data of copper rings were produced in 2018, the posterior distribution of consumption of copper rings in 2019 was combined with the actual consumption data in 2019, and when the posterior distribution of consumption of copper rings in 2019 is obtained, which can also be used as the prediction distribution function of consumption of copper rings in 2020. By analogy, the rolling prediction of copper ring consumption distribution can be realized.

5. Conclusion

In this paper, bayesian theory is used to effectively combine the prior distribution of spare parts consumed by equipment unit with sample data, and the prediction models of spare parts consumed by constant consumption with time are established respectively. On this basis, the prediction of spare parts consumed by constant consumption is carried out.

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