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The Risk Measurement of Shanghai Composite Index Based on Quantile GARCH Model

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Abstract—**With the continuous development of economic globalization, the issue about the risk of financial derivatives have grown considerably in recent years. When a financial crisis happened in a country, the global economy was severely affected by the catastrophic result. Value at risk has been a standard method for market regulators to measure the risk of market in the future. In this paper, we refer to the conditional quantile estimation for generalized autoregressive conditional heteroscedasticity models in Xiao and Koenker (2009), which has been proved to be a powerful model to measure value at risk. By showing the descriptive statistics of the logarithm returns of Shanghai Composite Index and results of ADF test and Ljung-Box test, we ensure the quantile GARCH model could be applied to Shanghai Composite Index. We apply the quantile GARCH model in Chinese stock market, which could be represented by Shanghai Composite Index. The empirical result demonstrate that the Quantile GARCH model works well in the Chinese stock market. Our paper could offer some important suggestions about risk for investors, financial institutions and stock regulators.**

Keywords—Value at risk; Quantile GARCH model; Chinese stock market; Financial market; Shanghai Composite Index

I. Introduction

The risk of financial market mainly comes from the uncertainty of economic situation, which would lead to the loss of investors and enterprises. Since the 19th century, international financial market has been in a state of frequent fluctuations. The bubble in Japanese economy, the subprime crisis in America and the financial crisis in Asia alarm the relevant regulators for financial market to strengthen the risk management of market. In the same time, the frequent occurrences of financial market crisis have attracted extensive attention from the financial researchers.

Since the establishment of the Chinese stock market in 1990, the stock market has attracted a large amount of capital and the market has been developing promptly. In the meanwhile, financial institutions and investors are taking huge risks while obtaining high returns, because of its highly speculative and highly volatile market characteristics. The research about risk measurement of financial market could not only guide investors and financial institutions to avoid huge loss in extreme case, but also provide suggestions for financial market regulators to control financial market risk.

Distributional information such as conditional quantiles and variances play an essential role in risk assessment. Evaluation of Value-at-Risk, as mandated in many current regulatory contexts, is explicitly a conditional quantile estimation problem. Quantile regression as introduced by Koenker and Bassett (1978) is well suited to estimating conditional quantiles. Most of previous studies about risk measurement focus on nonparametric models or semiparametric models. Mauser and Rosen(1999),Chen and Tang (2005), Pritsker(2006) focused on applying historical simulation method or Monte Carlo simulation method to calculate value at risk for stock market. Acerbi and Tasche (2001) proposed CVaR(Conditional VaR) model, and proof its consistency in risk measurement and other statistical properties. In recent years, quantile regression estimation for time-series models has gradually attracted more attention. Koenker and Zhao (1996) extended quantile regression to linear ARCH models. Manganelli and Engel (2004) proposed CAViaR model, which described the autoregressive relationship in value at risk. Xiao and Koenker (2009) proposed quantile GARCH model to predict the financial market risk in future, which has better performance than previous nonparametric models and parametric models(Khizar Qureshi, 2016; Robert Engle and Bassett, 1982; Bollerslev, 1986; Engel and NG, 1993).

On the basis of the risk measurement model of foreign markets, we calculate the value at risk for Chinese stock market and provide a powerful instrument for investors, financial institutions and Chinese market regulators.

II. Methodology

A. Regression Model

Quantile regression (Koenker and Basset, 1978) provides an insight into the relationship at different tails of the response distribution on the covariates. Let *y* be the response variable of interest and *x* be a *p*-dimensional vector of covariates. At any given quantile level τ, we define $Q_y(\tau|x)$ as the τth conditional quantile of y given x which is $Q_y(\tau|x) = F^{-1}(\tau) = inf\{t: F(t|x) \geq t\}$, where $F(\cdot|x)$ is the conditional distribution of *y* given *x*. We consider the following conditional quantile regression model as

$$Q_y(\tau|x) = x'\beta(\tau)$$

where $\beta(\tau)$ is a unknown *p*-dimensional regression coefficient. We could gain a more complete picture of the underlying

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structure of the conditional distribution of y by considering different quantiles (Koenker and Basset, 1978).

Besides, some scholars apply the quantile technique on the time series. Koenker and Zhao (1996) proposed an estimation procedure in quantile regression in ARCH models. Because the we assume the traditional ARCH model. Xiao and Koenker (2009) considered the estimation of the conditional quantiles for GARCH models for quantile regression. They proposed a simple and effective two-step approach for the estimation in the conditional quantile regression for GARCH models, and derived the asymptotic properties of the parameter estimators.

We can say y_t follows a linear GARCH(p,q) model, if

$$\begin{cases} y_t = \sigma_t \epsilon_t \\ \sigma_t = \beta_0 + \beta_1 \sigma_{t-1} + \beta_2 \sigma_{t-2} + \cdots + \beta_p \sigma_{t-p} \\ \quad +\gamma_1 |y_{t-1}| + \gamma_2 |y_{t-2}| + \cdots + \gamma_q |y_{t-q}|, \end{cases} \tag{1}$$

where ε_t are independent and identically distributed with mean 0 and unknown distribution function $F_\varepsilon(\cdot)$, $\beta_0 > 0$ and $\gamma_i > 0, i = 1, \cdots, q$.

Let $\mathcal{F}_{t-1}$ denotes the $\sigma-$field generated by $\{y_s, s \le t-1\}$, the conditional quantile GARCH function of y_t can be represented by

$$Q_{y_t}(\tau|\mathcal{F}_{t-1}) = x_t' \theta(\tau), \tag{2}$$

where

$$x_t = \left(1, \sigma_{t-1}, \cdots, \sigma_{t-p}, |u_{t-1}|, \cdots, |u_{t-q}|\right)',$$

$$\theta(\tau) = \left(\beta_0, \beta_1, \cdots, \beta_p, \gamma_1, \cdots, \gamma_q\right)' F_\varepsilon^{-1}(\tau).$$

In the following parts, we give a belief description on the conditional quantile estimation, $Q_{y_t}(\tau|\mathcal{F}_{t-1})$. In details, we introduce the two-step estimation method according to the quantile check loss function.

B. Two-step Estimation

Given the GARCH model (1) and (2), under some regularity assumption, we can obtain an ARCH(∞) representation for σ_t:

$$\sigma_t = a_0 + \sum_{j=1}^{\infty} a_j |y_{t-j}|. \tag{3}$$

For identification, we normalize $a_0 = 1$. Substituting (3) into (1) and (2), we have

$$y_t = \left(a_0 + \sum_{j=1}^{\infty} a_j |y_{t-j}|\right) \varepsilon_t. \tag{4}$$

Define $\alpha_j(\tau) = a_j Q_{\varepsilon_t}(\tau), j = 1,2, \cdots$, we can obtain

$$Q_{y_t}(\tau|\mathcal{F}_{t-1}) = \alpha_0(\tau) + \sum_{j=1}^{\infty} \alpha_j(\tau) |y_{t-j}| \tag{5}$$

Koenker and Xiao (2006) proposed that under regularity conditions, the coefficients a_j decrease geometrically and sieve approximation can simplify model (5). Let $m = \frac{3}{2} n^{\frac{1}{4}}$ denote the truncation parameter, (5) can be simplified into

$$Q_{y_t}(\tau|\mathcal{F}_{t-1}) = \alpha_0(\tau) + \sum_{j=1}^{m} \alpha_j(\tau) |y_{t-j}| + 0_p\left(\frac{m}{\sqrt{n}}\right). \tag{6}$$

In order to obtain a more robust estimation, first step estimation use the information over multiple quantile. We can estimate the m-th order quantile autoregression by

$$\tilde{\alpha}(\tau) = \arg\min_\alpha \sum_{t=m+1}^{n} \rho_\tau (y_t - \alpha_0 - \sum_{j=1}^{m} \alpha_j(\tau) |y_{t-j}|), \tag{7}$$

at given quantile $(\tau_1, \cdots, \tau_K)$, and obtain estimators $\alpha(\tau_k), k = 1, \cdots, K$.

Then we can obtain estimation of $a_j, j = 1, \cdots, m$ by

$$\tilde{a} = arg\, min_a (\tilde{\pi} - \phi(a))' A_n (\tilde{\pi} - \phi(a)) \tag{8}$$

where $a = (a_1, \cdots, a_m)'$, $q_k = Q_{\varepsilon_t}(\tau_k)$, $\tilde{\pi} = (\tilde{\alpha}(\tau_1)', \cdots, \tilde{\alpha}(\tau_K)')'$ A_n is a $K(m+1) \times K(m+1)$ positive defined matrix, and

$$\phi(a) = (q_1, \cdots, q_K)' \otimes a = [q_1, a_1 q_1, \cdots, a_m q_1, \cdots, q_K, a_1 q_K, \cdots, a_m q_K]'.$$

In empirical practice, we always let $A_n = I_{K(m+1)}$ and set $\tilde{a}_1 = 1$.

Thus σ_t can be estimated by

$$\tilde{\sigma}_t = \tilde{a}_1 + \sum_{j=1}^{m} a_j |y_{t-j}|. \tag{9}$$

Second step estimation focus on a certain quantile to estimate $Q_{y_t}(\tau|\mathcal{F}_{t-1})$. Combine (2) and (8), the $\theta(\tau)$ can be estimated by

$$\tilde{\theta}(\tau) = arg \min_\theta \sum_t \rho_\tau\big(y_t - \tilde{x}_t' \theta(\tau)\big), \tag{10}$$

where

$$\tilde{x}_t = \big(1, \tilde{\sigma}_{t-1}, \cdots, \tilde{\sigma}_{t-p}, |y_{t-1}|, \cdots, |y_{t-q}|\big)'.$$

Given $\tilde{\theta}(\tau)$ and $\tilde{x}_t$, the τ-th quantile of y_t can be estimated by

$$\tilde{Q}_{y_t}(\tau|\mathcal{F}_{t-1}) = \tilde{x}_t' \tilde{\theta}(\tau). \tag{11}$$

III. Data

The Shanghai Composite Index also known as SSE Index is a stock market index of all stocks (A shares and B shares) that are traded at the Shanghai Stock Exchange. The data are collected from Vander database, which contain the daily trade information from 2010.3.1-2016.12.30. Since we focus on the risk measurement of Chinese stock market, we calculate the logarithm returns y_t by

$$y_t = 100 * (ln p_t - ln p_{t-1}),$$

where p_t denotes the closing price of Shanghai Composite Index.

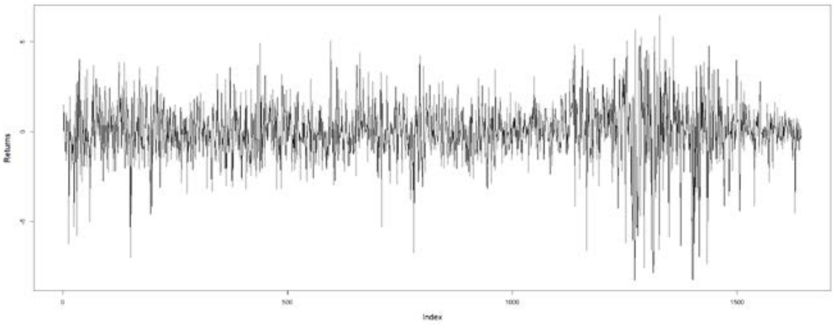

Figure 1. the logarithm returns of Shanghai Composite Index

In addition, we give the corresponding descriptive statistics of y_t and the results of ADF test and Ljung-Box test. The Tab I. ensures that quantile GARCH model could be applied to Shanghai Composite Index.

TABLE I: THE CORRESPONDING DESCRIPTIVE STATISTICS OF LOGARITHM RETURNS OF SSE INDEX

Statistics	**Values**
Sample size	1643
Mean	0.010
Median	0.048
Standard deviation	1.481
Maximun	5.763
Minimum	-8.490
25% quantile	-0.632
75% quantile	0.721
Skewness	- 0.637
Kurtosis	4.281
P value of J-B test	0.000
P value of ADF test	0.000
P value of Ljung-Box test	0.000

IV. Empirical Analysis

A. Coverage rate

If the significance level τ and the number of samples N are given, the times "break" the true value at risk should be $N \times \tau$. In this paper, we apply coverage rate to measure the performance of quantile GARCH model. Coverage rate denotes that based on time series data $\{y_t, t=1,\cdots,N\}$, the number of $y_t < \tilde{Q}_{y_t}(\tau|\mathcal{F}_{t-1}), t = 1,\cdots,T$. In mathematical version, coverage rate could be formulated by

$$coverage\ rate = \frac{1}{T}\sum_{t=1}^{T} I(y_t < \tilde{Q}_{y_t}(\tau|\mathcal{F}_{t-1}))$$

B. Empirical results

In this section, we calculate the value at risk by the quantile GARCH model and show it in the following figures, where black lines represent the logarithm returns of Shanghai Composite Index and red lines represent the τ-th value at risk. In this paper, we choose quantile GARCH(1,1) model for empirical study.

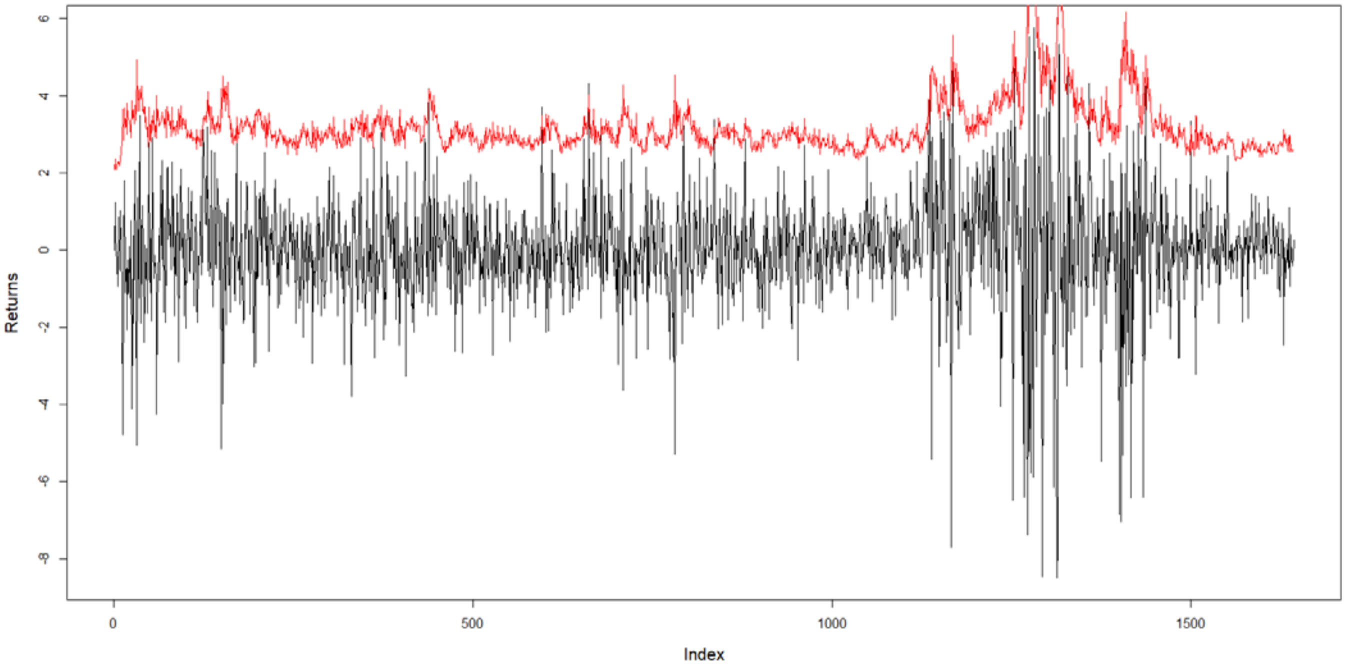

Figure 2. the 0.99-th value at risk of Shanghai Composite Index

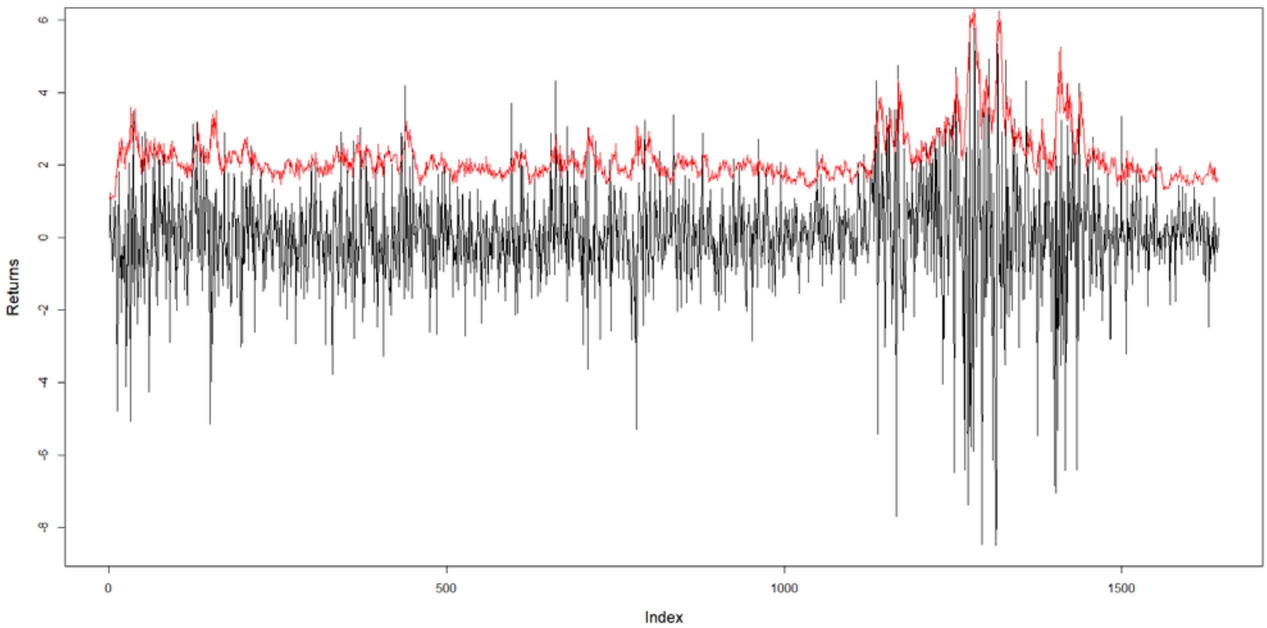

Figure 3 the 0.95-th value at risk of Shanghai Composite Index

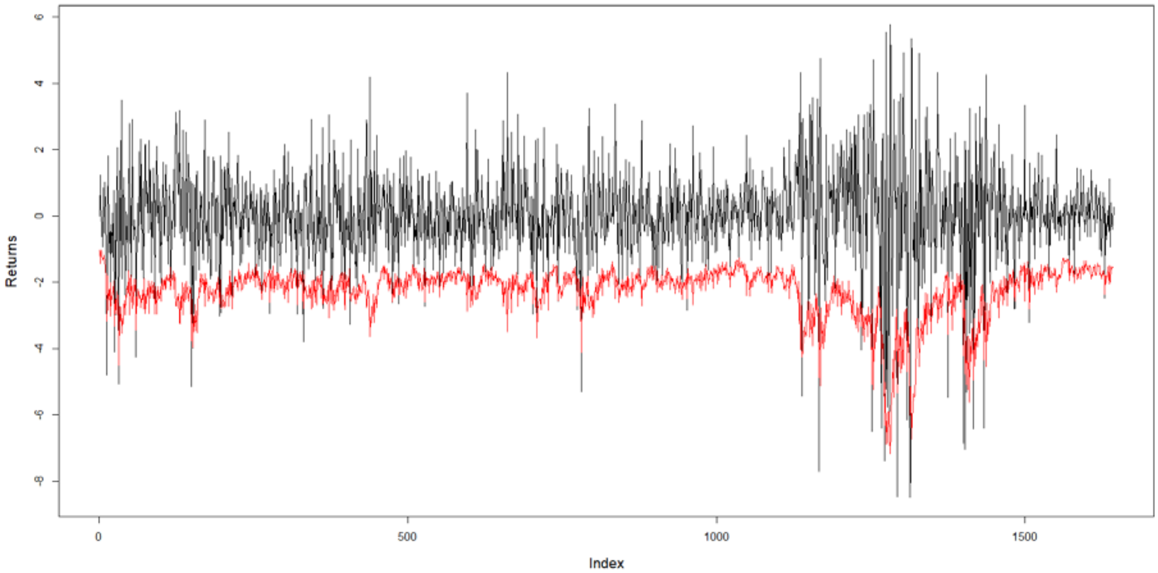

Figure 4. the 0.05-th value at risk of Shanghai Composite Index

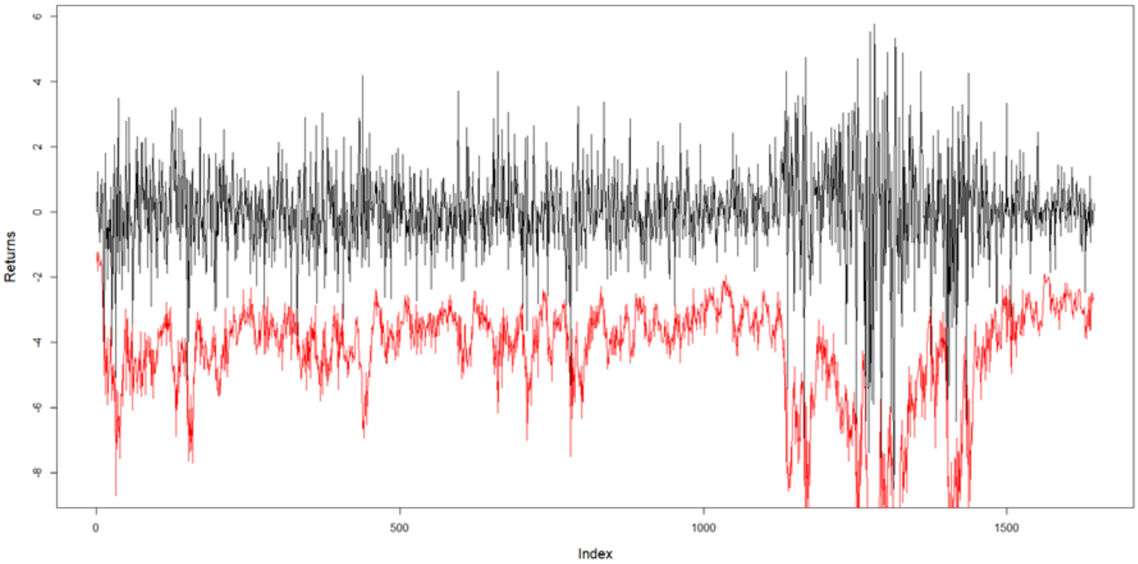

Figure 5. The 0.01-th value at risk of Shanghai Composite Index

In addition, we give the corresponding parameter estimation results and the coverage rate for each case.

TABLE II: PARAMETER ESTIMATION RESULTS AND THE COVERAGE RATE FOR EACH CASE

Significance level τ	Parameters	Value of parameter	P value	Coverage rate
0.99	$\beta_0(\tau)$	0.400	0.633	99.15%
	$\beta_1(\tau)$	1.611	0.002	
	$\gamma_1(\tau)$	0.116	0.525	
0.95	$\beta_0(\tau)$	0.198	0.685	94.88%
	$\beta_1(\tau)$	1.032	0.001	
	$\gamma_1(\tau)$	0.199	0.056	
0.05	$\beta_0(\tau)$	0.640	0.204	5.54%
	$\beta_1(\tau)$	-1.491	0.000	
	$\gamma_1(\tau)$	-0.197	0.061	
0.01	$\beta_0(\tau)$	1.330	0.124	0.97%
	$\beta_1(\tau)$	-3.022	0.000	
	$\gamma_1(\tau)$	-0.373	0.044	

From Tab II., we have following findings:

(1) The coverage rate is close to the significance level τ, which proof the quantile GARCH model make good performance in Shanghai Composite Index.

(2) The p values of $\beta_1(\tau)$ keep lower than 0.05, which ensure the $\beta_1(\tau)$ is significant in each case. This finding verifies the autoregression in financial stock index.

V. CONCLUSION

In this paper, we consider a risk measurement method of Chinese stock market. Following the research in Xiao and Koenker (2009), we summarize the estimation method and the implement of quantile GARCH model. We collect data from Vander database and calculate the logarithm returnsy_t. By calculate the descriptive statistics of the logarithm returns and results of ADF test and Ljung-Box test, quantile GARCH model could be applied to Shanghai Composite Index. The empirical analysis shows that quantile GARCH model make good performance in Shanghai Composite Index. Our paper could offer some important suggestions about risk for investors, financial institutions and stock regulators.

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